Transportmetrica, Vol. 4, No. 1 (2008), 19-32

EFFICIENCY LOSS OF PRIVATE ROAD WITH CONTINUOUSLY DISTRIBUTED VALUE-OF-TIME

FENG XIAO[1] AND HAI YANG[2]

Received 13 April 2007; received in revised form 17 September 2007; accepted 18 October 2007

As commercial and private provision of roads was used to finance modern road systems, many intriguing issues have emerged, such as the strategy of a private firm in determining road supply and pricing in a network and, more importantly, the resulting inefficiency when profit-oriented behavior substitutes for government regulation. This paper examines the likely bias of a monopoly market away from the social optimum under the more realistic assumption that each trip-maker has a unique value-of-time (VOT). The efficiency loss and the road capacity and/or toll set by a monopolist are investigated under different kinds of government regulatory regimes. Due to the informational difficulty of accurately valuating the VOT over all the trip-makers, the paper suggests that rate-of-return regulation may have attractive advantages for the regulatory authority.

KEYWORDS: Monopoly, social optimum, heterogeneousness, value-of-time, rate-of-return regulation

1. INTRODUCTION

Recently many countries have started massive highway franchising programs via so-called Build-Operate-Transfer (BOT) contracts. Under such a contract, the private sector not only operates but also finances the highway and in turn receives the revenue from road tolls for a long period. When the franchise ends, the road reverts to the government (Yang and Meng, 2000). It is generally believed that introducing private provision of public roads is an efficient way to expand modern road systems. Once the road provision becomes market driven, many issues must be addressed carefully, because the interests of the private sector are different from those of the public sector. From the viewpoint of private investors, profitability of a project is of great concern because their private firms are put at risk, while from the government side, it is meaningful to assess whether the construction of a road will give a positive welfare gain and also be profitable, so that it can be provided privately.

One of the major issues concerning a new private toll road under a BOT contract is the selection of its capacity and toll charge as well as the evaluation of the profitability to the private investor and the welfare gain of the society (Yang and Huang, 2005). If the private investor is free to select both capacity and toll charge, social welfare gain is not guaranteed as its major concern is its own profit; if both capacity and toll charge are governed by the government, social optimization can be realized, but the project may become less attractive to private investors due to inflexibility and may even become infeasible because of a negative profit. To avoid these two extreme situations, two kinds of government policies can be considered: one is that the government sets a (optimal) level of the toll charge but leaves the road capacity to be freely determined by the investor; the other is that the government fixes the (optimal) road capacity while the investor selects the toll charge freely. These two regimes may both lead to market failure, but the intrinsic reasons are different. One failure is associated with the fact that when profit is maximized with respect to capacity, the investment may not be optimally set;

[1] Department of Civil Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong SAR, P.R. China. Corresponding author (E-mail: fxiao@ucdavis.edu).

[2] Department of Civil Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong SAR, P.R. China.

while the other is caused by the allowance for the private investor to exploit its power over toll (Spence, 1975).

Similar issues have been studied by considering only homogeneous road users with a single value-of-time (VOT). Yang and Meng (2000) investigated the profitability or self-financing and social welfare gain of a single new toll road in a general network through numerical experiments. Later they showed that the self-financing theorem holds for each road in a full network individually and consequently for the network in aggregate provided each link is optimally priced and all capacities are optimized (Yang and Meng, 2002). Verhoef and Rouwendal (2004) addressed some implications of both the first-best and second-best congestion pricing for the applicability of the self-financing theorem using a numerical experimental approach, and they found that the volume-capacity ratio in the social optimum is identical for all links if they have the same marginal cost of capacity. This observation is made numerically rather than established theoretically on a general network with the assumption of a linear inverse demand function and traditional BPR (Bureau of Public Road) travel time function. De Borger and Van Dender (2005) analyzed a model with two substitute congestible facilities under three administrative regimes: (a) social optimum, (b) monopoly, and (c) duopoly in a sequential capacity-then-toll game. They show that equilibrium time delays are equal in regimes (a) and (b), but higher in regime (c). Namely, pricing and capacity choices under monopoly do result in the socially optimal service quality. This result is derived theoretically but with a linear inverse demand function. Xiao et al. (2007) obtain a more general result when studying the inefficiency of the oligopolistic equilibria of toll road competition. They proved that at both oligopolistic equilibria and social optimum, the volume-capacity ratio of each road remains unchanged and is only determined by the road's own unit construction cost. A one-shot game is considered where the road capacity and level of toll charge are determined simultaneously by each firm subject to the resulting traffic flow being in equilibrium.

A few existing works relaxed the limitation of a single VOT by considering different VOTs for different users. In Cheung et al. (1999), users are divided into a number of groups or classes according to their VOTs. Each group has a distinct group-specific demand function to characterize its trip rates. Users are assumed to minimize their individual generalized cost in choosing their routes on the basis of travel time and monetary cost. Various possibilities of profitability and welfare gain of a private toll road in a given network under various combinations of road capacity and toll charge are presented. They also compared and contrasted the outcomes with the case of a single average VOT, and investigated how the VOT distribution affects the traffic flow and profit forecasts. Yang et al. (2002) further examined the impact of user heterogeneity on the profitability and social welfare gain of new toll roads. Mayet and Hansen (2000) developed a model for steady-state congestion pricing in which the VOT has a continuous distribution. They mainly focused on the difference of optimal tolls when the social welfare function is measured in money and time units respectively. A relevant earlier work by Spence (1975) dealt with market problems that arise when a monopoly sets some aspect of product quality as well as price, and makes some interesting findings. For example, if users value product quality equally (corresponding to homogeneous travelers with identical VOT), the profit-maximizing and socially optimal quality levels coincide (in the case of congested highways considered here, "product quality" just corresponds to congestion delay). Spence also discussed the advantages of the rate-of-return regulation for the considered market model.

This paper investigates the outcomes (profits and social welfare gains, etc.) and the corresponding inefficiency under three alternative regimes. The foregoing models are extended by considering continuously distributed VOT of the trip-makers instead of a single one. In comparison with the model used in Mayet and Hansen (2000), a more general complicated situation is considered, involving both toll charge and capacity as variables. By fixing one variable, the difference of the other one is examined between the profit maximizing and socially optimal situations. In contrast with most existing results in the literature where profit-oriented strategies always lead to higher toll charges and lower investments, it is found that, when considering heterogeneous trip-makers, one can not simply distinguish which one of the private investor and the government will give a higher toll charge or capacity, since their toll and capacity selection depends on how the valuation of the time is distributed among the whole population.

The paper is structured as follows: first, the model used throughout the analysis is introduced; then three different kinds of regimes are examined in which choice of either toll or capacity or both are considered. The efficiency loss of each regime is analyzed by comparing the monopoly choice and social optimum; in the last part, the advantage of rate-of-return regulation is discussed in the situation of information deficiency.

2. THE MODEL

Here we consider a simplified model to study the steady-state congestion pricing problem. Suppose in a general network there is an amount of fixed demand *P* from node *A* to node *B*. To shorten the travel time between *A* and *B*, the government decides to build a highway directly connecting the two nodes. Because of financing problem, the government has to use a BOT scheme by providing a private firm the franchise to build and operate the highway. Now the population of *P* trip-makers have two choices: they may use the new highway, in which case they are charged a toll τ. Their travel time follows a function $T(v, y)$, where *v* is the flow of trip-makers on the highway and *y* is the highway capacity. Without loss of generality, we assume that $\partial T/\partial v > 0$, $\partial^2 T/\partial v^2 > 0$ and $\partial T/\partial y < 0$; or they can take a network of roads free of charge with a fixed travel time t_0. It is assumed that the road network is large enough and never congested, and it is further assumed that $t_0 > T(0)$, where $T(0)$ is the free flow travel time of the highway, so that the tolled highway will certainly attract some users. A schematic representation of the situation modeled is given in Figure 1.

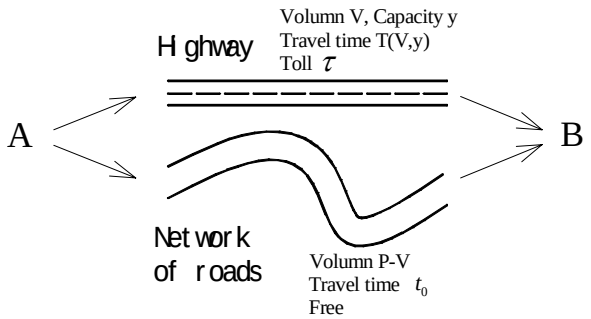

FIGURE 1: Model set-up

The capacity y and toll charge τ of the highway are the only two variables to be determined by the government or the private firm subject to the resulting traffic flow distribution being in a deterministic user equilibrium. The volume/capacity ratio, γ, is a representation of the level of service on the highway. Instead of just a single VOT for the whole population P, a more realistic situation that each trip-maker has a unique VOT is considered here. The distribution of VOT across the population is characterized by a continuous function $\beta(v)$. If the population is ordered in decreasing order of VOT, $\beta(v)$ gives the VOT of the vth trip-maker. Assume that $\beta(v)$ is continuous and differentiable in its domain of definition, then $\beta'(v) < 0$ (see Figure 2).

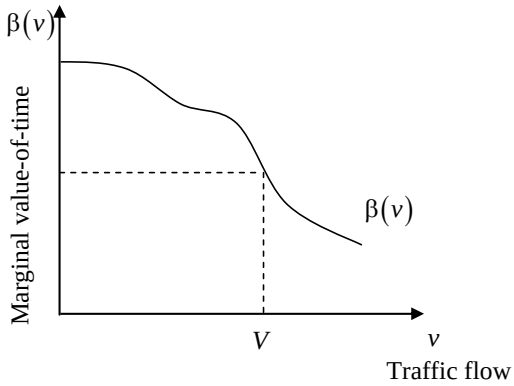

FIGURE 2: Continuously distributed VOT

We are ready to obtain the relationship between $\beta(v)$ and F (the cumulative function of VOT) as follows:

$$\beta(v) = F^{-1}\left(1 - \frac{v}{P}\right). \tag{1}$$

We also have the following relationship between $\beta'(v)$ and f (the probability density function of VOT):

$$\beta'(v) = -\frac{1}{Pf\left[\beta(v)\right]} < 0. \tag{2}$$

Trip-maker v will choose the highway if $\tau + \beta(v) \cdot T(V, y) \le \beta(v) \cdot t_0$, where V is the number of trip-makers using the highway. The full price that the vth trip-maker is willing to pay to use the highway is

$$D(v) = \beta(v) t_0. \tag{3}$$

The benefit of the vth trip-maker using the highway can be defined as $\beta(v) \cdot t_0 - \tau - \beta(v) \cdot T(V, y)$. For any equilibrium with $V < P$, since $t_0 > T(V, y)$, the benefit of the Vth trip-maker, the highway user with the lowest VOT and therefore the lowest willingness to pay for using the highway, is zero. Thus we have the following important formula

$$\beta(V) t_0 - \beta(V) T(V, y) - \tau = 0, \tag{4}$$

or

$$\tau = \beta(V) \cdot \left[t_0 - T(V, y) \right]. \tag{5}$$

Equation (4) implicitly defines V as a function of τ and y. Utilizing the implicit function theorem, we can obtain the two derivatives V_τ and V_y,

$$V_\tau = \frac{1}{(t_0 - T)\beta' - \beta T_V}, \tag{6}$$

$$V_y = \frac{\beta T_y}{(t_0 - T)\beta' - \beta T_V}. \tag{7}$$

Because $\beta' < 0$, $T_V > 0$ and $T_y < 0$, we have $V_\tau < 0$ and $V_y > 0$.

3. EFFICIENCY LOSS WITH CONTINUOUSLY DISTRIBUTED VOT

In this section the outcomes of three regimes and the corresponding inefficiency are investigated. All the three regimes lead to market failure, but for different intrinsic reasons.

3.1 Monopoly with fixed toll

In this sequential game, toll is given by the government first and the capacity is selected by the private firm whose concern is profit maximization. The game follows a two-stage process: at the first stage, the government gives a toll level; at the second stage, the firm decides the capacity level to invest on the construction of the highway.

We define τ^* as the toll level given by the government and $\bar{y}$ as the capacity provided by the private firm. The firm tries to maximize its profit given by the function with respect to *y*

$$\max \pi(y) = \tau^* \cdot V\left(\tau^*, y\right) - I(y). \tag{8}$$

Taking the first-order derivative we have

$$\pi'(y) = V_y \tau^* - I' = 0. \tag{9}$$

Thus we have

$$\tau^* = \frac{I'}{V_y}. \tag{10}$$

On the other hand, given the fixed toll level τ^* the social welfare is maximized with respect to *y* by the following function

$$W(V, y) = \left(t_0 - T(V, y)\right) \int_0^V \beta(v)\mathrm{d}v - I(y) = \tau^* \frac{1}{\beta} \int_0^V \beta(v)\mathrm{d}v - I_y \ . \tag{11}$$

The first-order optimality condition is

$$\frac{\partial W}{\partial V} = \tau^* \left(1 - \frac{\beta'}{\beta^2} \int_0^V \beta(v)\mathrm{d}v \right) V_y - I' = 0. \tag{12}$$

Thus

$$\tau^* = \frac{I'}{V_y}\frac{1}{1-\frac{\beta'}{\beta^2}\int_0^V \beta(v)\mathrm{d}v}. \tag{13}$$

If the private firm chooses the same capacity level as the optimal capacity level with fixed toll τ^*, then the traffic flows must also be the same on the highway. Since $\beta' < 0$, from equations (10) and (13) we readily have

$$\tau^* < \frac{I'}{V_y} = \tau^*, \tag{14}$$

which conflicts. Thus the capacity will not be optimally set under monopoly and the traffic flows differ. To simplify the discussion, the following widely used assumption is made:

ASSUMPTION 1 Constant return to scale in road construction: *There are constant returns to scale in road construction, namely,* $E_I^y = 1$ *or* $I(y) = \kappa y$, *where* E_I^y *is the elasticity of investment cost, I, with respect to output capacity, y, and* $I(y)$ *is the link construction cost function of the highway;* κ *denotes the unit capacity cost.*

If we define the optimal capacity level with fixed toll τ^* as y^S, with this assumption, from equations (10) and (13) we have

$$0 < V_y\left(\tau^*, \bar{y}\right) < V_y\left(\tau^*, y^S\right). \tag{15}$$

Inequality (15) states that whether the firm selects a lower or higher capacity than the government is actually determined by the marginal effect of capacity on traffic flow. If the marginal influence of capacity on traffic flow is diminishing, the profit-maximizing monopoly company will set a capacity lower than the socially optimal one, and conversely. If all the trip-makers have exactly the same value-of-time, i.e. $\beta' = 0, \forall v \in (0, P)$, the profit-maximizing behavior exactly obtains the same result with the optimum. Here we summarize those partial effects into the following Proposition 1.

PROPOSITION 1: *With Assumption 1, for any given toll charge, there is a difference in capacity level between the monopoly market and the social optimum if the trip-makers are heterogeneous, which means that the monopoly market is inefficient; the firm undersupplies capacity relative to the optimum when the marginal influence of capacity on traffic flow is diminishing, and conversely, the firm oversupplies the capacity.*

3.2 Monopoly with fixed capacity

Most studies of capacity-and-price competition adopt a two-stage game framework, in which capacities are chosen in the first stage and prices in the second. The two-stage sequential game is natural in the context of toll roads since adjusting road capacity is more costly and time-consuming than adjusting tolls. Indeed, unless tolls are heavily regulated it is easy to adjust tolls using electronic toll collection technology. Thus it could be more reasonable if we consider the sequential game, in which the level of construction capacity is given by the government first and the toll charge level is

selected by firm for profit maximization. Similarly, the game also follows a two-stage process.

We define y^* as the capacity level given by the government and $\bar{\tau}$ as the toll provided by the private firm. Because the capacity of the highway is fixed, the private firm can maximize its profit given by the following function only with respect to τ

$$\max \pi(\tau) = \tau \cdot V\left(\tau, y^*\right) - I\left(y^*\right). \tag{16}$$

Taking the first-order derivative we have

$$\pi'(\tau) = V + V_\tau \tau = V + \tau \frac{1}{\frac{\beta'}{\beta}\tau - \beta T_V} = 0. \tag{17}$$

Thus

$$\bar{\tau} = \frac{\beta T_V V}{1 + \frac{\beta'}{\beta} V}. \tag{18}$$

In contrast, the social welfare is maximized with respect to V by the following function

$$W\left(V, y^*\right) = \left(t_0 - T\left(V, y^*\right)\right) \int_0^V \beta(v)\mathrm{d}v - I\left(y^*\right). \tag{19}$$

The first-order condition is

$$\frac{\partial W}{\partial V} = \tau - T_V \int_0^V \beta(v)\mathrm{d}v = 0. \tag{20}$$

Thus the optimal toll level τ^s given the fixed capacity y^* is

$$\tau^* = T_V \int_0^V \beta(v)\mathrm{d}v = V T_V \bar{\beta}. \tag{21}$$

If we suppose the toll on the highway remains unchanged under both monopoly and at social optimum, then the traffic flows will also be the same, which means, from equations (18) and (21) that

$$\frac{\bar{\tau}}{\tau^s} = \frac{\beta}{\bar{\beta}\left(1 + \frac{\beta'}{\beta} V\right)} = 1. \tag{22}$$

Here the term $\beta' V / \beta$ is actually the elasticity of VOT with respect to the ordered trip-maker at V, which can be expressed by E_β^V. Thus equation (22) can be written as

$$\frac{\bar{\tau}}{\tau^s} = \frac{\beta}{\bar{\beta}\left(1 - \left|E_\beta^V\right|\right)} = 1. \tag{23}$$

Generally, equation (23) cannot hold, which causes different flows in the monopoly and social optimum cases. When $\left|E_\beta^V\right|$ is small enough, $\bar{\tau} / \tau^s < 1$, the monopoly firm tends to set a relatively lower toll on the highway than the government, and conversely,

set a relatively higher toll when $\left|E_{\beta}^{V}\right|$ is relatively large. Again, if all the trip-makers have exactly the same VOT, i.e. $\beta' = 0, \forall v \in (0, P)$, then it can be easily found that the profit-maximizing toll is exactly the same as the social optimum. We can summarize the partial effects into the following Proposition 2.

PROPOSITION 2: *For any given capacity, there is a difference in toll charge level between the monopoly market and the social optimum if the trip-makers are heterogeneous, which means that the monopoly market is inefficient; the firm undercharges the toll relative to the optimum when the absolute value of the elasticity of VOT is very small, and conversely the firm overcharges the toll.*

Here two examples are provided. In the first example, suppose that VOT follows a uniform distribution: $\beta(v) = b - av$, $a > 0, b > 0$, then it follows that

$$\frac{\bar{\tau}}{\tau^s} = \frac{(b - aV)^2}{(b - aV)^2 - abV} > 1. \tag{24}$$

Second, if VOT follows an exponential distribution, $\beta(v) = b - a\ln v$, $a < 0, b > 0$, then it follows that

$$\frac{\bar{\tau}}{\tau^s} = \frac{(b - a\ln V)^2}{(b - a\ln V)^2 - a^2} > 1. \tag{25}$$

Thus for uniformly or exponentially distributed VOT, the monopoly firm will always set a toll higher than the optimum.

However, one can easily find a counterexample to demonstrate the occurrence of the opposite situation. Suppose $\beta(v) = a + b/v$, $a > 0, b > 0$, then,

$$\frac{\bar{\tau}}{\tau^s} = \frac{a^2 V + 2ab + \dfrac{b^2}{V}}{a^2 V + ab\ln V}. \tag{26}$$

Clearly, when V is large enough, $\bar{\tau}/\tau^s < 1$.

3.3 Simultaneous game with both toll and capacity as decision variables

In contrast with the above two kinds of sequential games, in this section two highway operators, the private firm and the government, are assumed to choose both of the toll and capacity simultaneously. To obtain insightful results, the following widely used assumption is made.

ASSUMPTION 2 Homogeneous of degree zero link travel time function: *The link travel time function $T(V, y)$ is homogeneous of degree zero in both link flow V and link capacity y.*

This assumption is equivalent to assuming that the speed of traffic on the road is dependent only on the volume-capacity ratio $\gamma = V/y$ of the road, that is,

$T(V,y)=T(\gamma)$, such as the BPR (Bureau of Public Roads) type of function used widely. With Assumption 2, it is natural to assume that $T(\gamma)$ is convex with respect to γ. The two derivatives V_τ and V_y become

$$V_\tau=\frac{1}{(t_0-T)\beta'-\beta T'\frac{1}{y}}, \tag{27}$$

$$V_y=\frac{-\beta T'\gamma}{(t_0-T)y\beta'-\beta T'}. \tag{28}$$

Profit maximization yields the following two first-order optimality conditions

$$\pi_y=V_y\cdot\tau-I'=\frac{-\beta^2T'\gamma(t_0-T)}{(t_0-T)y\beta'-\beta T'}-I'=0, \tag{29}$$

$$\pi_\tau=V+V_\tau\tau=V+\frac{y\beta(t_0-T)}{(t_0-T)y\beta'-\beta T'}=0. \tag{30}$$

We define γ^* and $\overline{\gamma}$ as the volume-capacity ratios under the control of the government and the private firm, respectively. From equations (29) and (30) the following two results are obtained

$$T'(\overline{\gamma})\overline{\gamma}^2=\frac{I'}{\beta(\overline{V})}, \tag{31}$$

$$\overline{\tau}=T'(\overline{\gamma})\overline{\gamma}\frac{\beta^2(\overline{V})}{\beta(\overline{V})+\overline{V}\beta'(\overline{V})}, \tag{32}$$

$$\overline{\tau}=\frac{I'}{\overline{\gamma}}\frac{1}{1+\frac{\overline{V}\beta'(\overline{V})}{\beta(\overline{V})}}=\frac{I'}{\overline{\gamma}}\frac{1}{1-\left|E_\beta^{\overline{V}}\right|}. \tag{33}$$

However, for the social optimum, when the social welfare is measured in units of money

$$W(\tau,y)=\int_0^V D(v)\mathrm{d}v-\int_0^V\beta(v)\cdot T(\gamma)\mathrm{d}v-\kappa y=\left[t_0-T(\gamma)\right]\int_0^V\beta(v)\mathrm{d}v-I(y). \tag{34}$$

Taking the first-order derivatives yields

$$W_y=\frac{1}{y}T'\gamma\int_0^V\beta(v)\mathrm{d}v-I'=0, \tag{35}$$

$$W_V=\tau-\frac{1}{y}T'\int_0^V\beta(v)\mathrm{d}v=0. \tag{36}$$

From equations (35) and (36) the following two results are obtained

$$T'\left(\gamma^*\right)\gamma^{*2} = \frac{I'}{\frac{1}{V^*}\int_0^{V^*}\beta(v)\mathrm{d}v}, \tag{37}$$

$$\tau^* = T'\left(\gamma^*\right)\gamma^*\left[\frac{1}{V^*}\int_0^{V^*}\beta(v)\mathrm{d}v\right] = T'\left(\gamma^*\right)\gamma^*\overline{\beta}, \tag{38}$$

$$\tau^* = \frac{I'}{\gamma^*}. \tag{39}$$

By comparing equations (31) and (37), the following proposition is obtained.

PROPOSITION 3: *With Assumptions 1 and 2, there is a difference in toll charge level between the monopoly market and the social optimum if the trip-makers are heterogeneous, which means that the monopoly market is inefficient; and the monopoly market is fully efficient when the commuters are homogeneous.*

Proof: Suppose monopoly market has the same toll and capacity levels with socially optimal solution, then the traffic flow must be the optimal flow as well, that is, $\overline{V} = V^* = V$. And obviously, $\overline{\gamma} = \gamma^*$. However, with Assumption 1 and from equations (31) and (37) it follows that

$$T'(\overline{\gamma})(\overline{\gamma})^2 = \frac{\kappa}{\beta(V)} > \frac{\kappa}{\frac{1}{V}\int_0^V \beta(v)\,\mathrm{d}v} = T'\left(\gamma^*\right)\left(\gamma^*\right)^2, \tag{40}$$

which implies $\gamma^* < \overline{\gamma}$. This conflicts with the assumption that $\overline{\gamma} = \gamma^*$.

The proof of the second part is straightforward. When β = constant, $\beta'(V) = 0$. Since $T(\gamma)$ is convex with respect to γ, from equations (31) and (37) it follows that

$$\beta(V) = \frac{1}{V}\int_0^V \beta(v)\,\mathrm{d}v \quad \Rightarrow \quad \overline{\gamma} = \gamma^*. \tag{41}$$

Also, from equations (39) and (33), $\tau^* = \overline{\tau}$. Thus the two situations coincide.■

It is hard to compare the toll and capacity levels under monopoly and at social optimum, since the overall performance of the monopolist is the conjunction of the two partial effects in Sections 3.1 and 3.2. The firm with market power deviates from the optimum in either one or both of the two aspects: on the one hand, the firm intends to give a higher toll than the optimum, which causes a lower traffic on the highway. On the other hand, the firm sets *y* lower than the socially optimal capacity.

COROLLARY 1: *With Assumptions 1 and 2, the monopoly market can not have both a lower toll and a higher capacity compared with the social optimum.*

The source of the potential market failure is located on the different VOT for the trip-makers. An intuitive explanation can be given here. Consider a firm which is contemplating a small increase in the investment for the construction of capacity. The

increase in capacity will increase costs, say by ΔI. It will also increase revenues. The increase in capacity reduces the highway travel time and thus increases the unit monetary benefit of the highway to the marginal trip-maker (the one who is just willing to pay for the toll) by $\Delta\tau(v)$, where v is the number of trip-makers. The firm will increase revenues by $v\Delta\tau(v)$. This increase is desirable for the firm if $v\Delta\tau(v) > \Delta I$. On the other hand, the capacity increase is desirable for the society if the total benefits $-\Delta T(V)\int_0^V \beta(v)dv$ exceeds the cost ΔI. Comparing the two situation, it is found that the social benefits coincide with the increase in the revenues of the firm only if the marginal benefit of the trip-maker is equal to the average, that is, when $-(1/V)\Delta T(V)\int_0^V \beta(v)dv = \Delta\tau(v)$, or $-\Delta T(V)\bar{\beta} = \Delta\tau(v)$. Here, for the demand to remain unchanged, $\Delta\tau(v)$ and $\Delta T(V)$ have the relationship: $\Delta\tau(v) = \beta(V)\Delta T(V)$. Thus, only when $\bar{\beta} = \beta$, the social benefits correspond to the increase in the revenues of the firm. But according to the definition of $\beta(v)$, $\bar{\beta}$ can not be less than β and they are equal only if all the trip-makers have the same VOT.

Now the v/c ratio on the highway is examined. From equations (5), (31) and (33) it follows that

$$\frac{t_0 - T(\bar{\gamma})}{T'(\bar{\gamma})\bar{\gamma}} = \frac{1}{1 - \left|E_{\beta}^{\bar{V}}\right|}, \tag{42}$$

where $\bar{V}$ is the total amount of traffic on the highway under the control of private firm. Similarly, from equations (5), (37) and (39)

$$\frac{t_0 - T(\gamma^*)}{T'(\gamma^*)\gamma^*} = \frac{\bar{\beta}}{\beta}. \tag{43}$$

Let $g(\gamma) = (t_0 - T(\gamma))/(T'(\gamma)\gamma)$. Obviously $g(\gamma)$ is decreasing with respect to γ. And

$$\frac{g(\bar{\gamma})}{g(\gamma^*)} = \frac{\beta(V^*)}{\bar{\beta}(V^*)}\frac{1}{1 - \left|E_{\beta}^{\bar{V}}\right|}. \tag{44}$$

Similarly, like the discussion in Section 3.2, it is found that whether the v/c ratio of the highway controlled by the firm is higher or lower relative to the optimum depends on the absolute value of the elasticity of VOT at $\bar{V}$.

4. RATE-OF-RETURN REGULATION

When government controls both toll charge and construction capacity, the profit realized for the firm is

$$\pi = \tau^* \cdot V(\tau^*, y^*) - I(y^*) = \frac{\kappa}{\gamma^*}V^* - I(y^*) = 0. \tag{45}$$

However, the regulatory authority is facing an informational problem. It is difficult to know the distribution of VOT. Therefore it is almost impossible to determine the optimal toll and capacity levels. Facing the informational difficulty, it is shown here that even VOT is continuously distributed, rate-of-return regulation may continue to be a desirable policy choice by the government.

The rate-of-return constraint takes the form

$$\frac{\tau V}{I} \le s\,, \tag{46}$$

or

$$\pi \le (s-1) I(y)\,, \tag{47}$$

which means that an upper-bound of the revenue of the private firm per unit construction cost is given by *s*. Since the toll charge level on the highway τ, the traffic flow on the highway *V* and the total construction cost *I* can all be observed, the only question for the government is to decide the upper-bound of the rate-of-return *s*. Now it is shown that the imposition of an upper-bound on the rate-of-return is equivalent to imposing a lower-bound on constructed capacity.

Assuming that the profit of the private firm is concave with respect to capacity and has a maximum at point A, from equation (45) it is known that the profit curve intersects *x*-axis at point $C\left(y^*, 0\right)$. The curve, $f(y) = (s-1) I(y)$, is drawn in Figure 3. Obviously, when $s > 1$, $f'(y) = (s-1) I' \ge 0$, the rate-of-return constraint is a positively sloped function in profit-capacity space. In general, the position of the rate-of-return constraint will depend on the allowed rate-of-return, *s*. When $1 \le s \le \bar{s}$, as *s* increases profit increases fast and less investment on capacity is induced. When $s \ge \bar{s}$, the capacity will remain at a minimum of $\bar{y}$. From equations (33), (8) and (47), it follows that

$$\pi_{\max} = \frac{\bar{I}_y \bar{y}}{1 - \left| E_\beta^{\bar{V}} \right|} - \bar{I} = (\bar{s} - 1)\bar{I}\,. \tag{48}$$

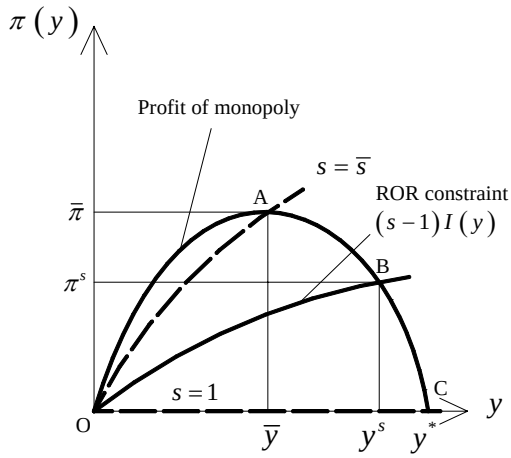

FIGURE 3: Profit maximization with a rate-of-return constraint

Thus $\bar{s}$ is obtained from equation (48) as

$$\bar{s} = \frac{\bar{I}_y \bar{y}}{\bar{I}\left(1 - \left|E_\beta^{\bar{V}}\right|\right)}. \tag{49}$$

With Assumption 1, $\bar{I}_y \bar{y} = \bar{I}$, $\bar{s}$ can be further simplified into

$$\bar{s} = \frac{1}{1 - \left|E_\beta^{\bar{V}}\right|}. \tag{50}$$

That is to say, any upper-bound of rate-of-return $s \le \left(1 - \left|E_\beta^{\bar{V}}\right|\right)^{-1}$ can drive the private firm to increase its investment on capacity construction when maximizing its own profit and thus to reduce the inefficiency loss under monopoly.

In this case when the government forces the rate-of-return to be 1, then optimal investment will be obtained under monopoly with a zero profit gain. The government may also impose a $s < 1$ on the private firm, the capacity will be oversupplied by the private firm. But as a result, the profit of the private firm becomes negative so that actually the whole project is infeasible for the private firm to be carried out.

5. CONCLUSION

This study shows how the valuation of time by trip-makers can influence the monopolist's selection of price and investment for infrastructure construction. Even if the demand of the market is inelastic, the profit-maximizing behavior of the private firm can not be fully efficient because of the VOT diversity. Though ideally regulation can manage toll-investment tradeoffs by confronting the firm, on behalf of the social welfare, in fact the optimal toll and capacity can hardly be determined. The difficulty lies on several aspects: first, computing the optimal toll and capacity requires knowledge of the value attached to travel time by the full range of trip-makers; second, the efficiency loss is still inevitable when only simple toll constraint or capacity constraint is adopted; third, from the viewpoint of private firm, profitability of a project has to be guaranteed because risks of investment in roads are very high compared to alternative investment options. Thus the rate-of-return regulation is brought forward, instead of simply controlling toll or capacity or both. The rate-of-return regulation has some merit as a second-best strategy, such as the information requirement is reduced, and the rate of substitution between toll and capacity is easily to be controlled through only one decision variable.

Currently our study is only limited to a single toll road that competes with a free-access road, it would be meaningful to extend our results to more general cases such as multiple parallel toll roads under different ownerships (all private toll roads or private vs. public toll roads). Moreover, the analytical results based on simple parallel network could also be extended for more general cases, like cordon-based pricing or second-best pricing on a general network (Zhang and Yang, 2004; Akiyama and Okushima, 2006).

ACKNOWLEDGEMENTS

The research described here was supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. HKUST6215/06E).

REFERENCES

Akiyama, T. and Okushima, M. (2006) Implementation of cordon pricing on urban network with practical approach. Journal of Advanced Transportation, 40, 221-248.

Cheung, W.M., Yang, H. and Tang, W.H. (1999) Modeling private toll roads with heterogeneous users. Proceedings of the Fourth Conference of the Hong Kong Society for Transportation Studies, 4 December, Hong Kong, pp. 189-198.

De Borger, B. and Van Dender, K. (2005) Prices, capacities and service quality in a congestible Bertrand duopoly. ERSA conference papers (ersa05p221), European Regional Science Association.

Mayet, J. and Hansen, M. (2000) Congestion pricing with continuously distributed values of time. Journal of Transport Economics and Policy, 34, 359-370.

Spence, M. (1975) Monopoly, quality, and regulation. Bell Journal of Economics and Management Science, 6, 417-429.

Xiao, F., Yang, H. and Han, D.R. (2007) Competition and efficiency of private toll roads. Transportation Research Part B, 41, 292-308.

Verhoef, E.T. and Rouwendal, J. (2004) Pricing, capacity choice and financing in transportation networks. Journal of Regional Science, 44, 405-435.

Yang, H. and Huang, H.J. (2005) Mathematical and Economic Theory of Road Pricing. Elsevier, Oxford.

Yang, H. and Meng, Q. (2000) Highway pricing and capacity choice in a road network under a build-operate-transfer scheme. Transportation Research Part A, 34, 207-222.

Yang, H. and Meng, Q. (2002) A note on highway pricing and capacity choice under a build-operate-transfer scheme. Transportation Research Part A, 36, 659-663.

Yang, H., Tang, W.H., Cheung, W.M. and Meng, Q. (2002) Profitability and welfare gain of private toll roads in a network with heterogeneous users. Transportation Research Part A, 36, 537-554.

Zhang, X. and Yang, H. (2004) The optimal cordon-based network congestion pricing problem. Transportation Research Part B, 38, 517-537.