Transportmetrica, Vol. 1, No. 1 (2005), 47-63

STRATEGIES FOR ROAD NETWORK DESIGN OVER TIME: ROBUSTNESS UNDER UNCERTAINTY

W.Y. SZETO[1] AND HONG K. LO[2]

Received 22 March 2004; received in revised form 11 June 2004; accepted 15 June 2004

Planning road network improvements over a long horizon often faces uncertainty. Under resource constraints, the government needs to carefully select a design strategy so as to cope with uncertainty and achieve its objective(s). Through sensitivity analyses, this paper discusses the social and financial aspects of three government's network design strategies, namely government-as-the-provider (GP), monopoly market (MM), and competitive market (CM). The first is the government acts as the sole toll road operator who provides the infrastructure but collects just enough tolls to recover costs. The second is the private sector builds all the toll roads based on the build-operate-transfer scheme and collects tolls in a monopoly manner. The third is the government allows multiple operators to build different toll roads but collect tolls under the build-operate-transfer scheme in a competitive manner. The results show that even though the GP strategy would produce the best total discounted consumer surplus (TDCS), maintaining cost recovery might be a problem. On the other hand, the MM strategy would mostly likely be profitable to the operators, at the expense of producing a much lower TDCS. It appears that the CM strategy would maintain a good balance between TDCS and profitability under uncertainty. This finding is consistent with the current trend of privatizing government services to multiple companies.

KEYWORDS: Time-dependent transport network design, network improvements financing, build-operate-transfer transport projects

1. INTRODUCTION

Transport infrastructure development is very active in many parts of the world especially in Asia. Traditionally, this analysis belongs to the discipline of transport network design. A recent review on models and solution approaches on this discipline is provided by Yang and Bell (1998). This discipline can broadly be classified into two approaches: Without and with the considerations of the time dimension. Past efforts (e.g. LeBlanc, 1975; Boyce and Janson 1980; Marcotte, 1986; Chen and Alfa, 1991; Friesz et al., 1993; Davis, 1994; Yang and Meng, 2000) relied on the approach that does not consider the temporal aspect of network improvements. In reality, transport infrastructure projects are not a one-time event, but will be in operation far into the future. It is important to incorporate the time dimension into the analysis.

Recently, some efforts (e.g. Lo and Szeto, 2003, 2004a,b,c) introduce the time dimension to network design. This approach considers not only the time-dependent user-equilibrium assignment with elastic demands but also time-dependent travel demands and the gradually upgraded network during the planning horizon. With this consideration, one can begin to design for the optimal project initiation time, phasing, toll structure, and financial arrangement over the planning horizon. In particular, Lo and Szeto (2004c) examine and compare three government network design strategies: namely government-as-the-provider (GP), monopoly market (MM), and competitive market (CM). The first is the government acts as the sole toll road operator who provides the infrastructure but collects just enough tolls to recover costs. The second is the private sector, who acts as a monopoly, builds all the toll roads based on the build-operate-

[1] Department of Civil, Structural, and Environmental Engineering, Trinity College, University of Dublin, Dublin 2, Ireland. Corresponding author (E-mail: szetow@tcd.ie).

[2] Department of Civil Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong SAR, P.R. China.

transfer (BOT) scheme. The third is the government allows multiple operators to build different toll roads based on the BOT scheme and collect tolls in a competitive manner. However, their analysis assumes that the parameters are known accurately. In reality, the forecasted population growth and travel demands are subject to uncertain future land use patterns and/or rail system additions unknown at the time of the planning. Moreover, maintenance and construction cost estimates fluctuate over time; the value of time of travelers, interest rates, inflation rates and the economic condition also cannot be forecasted perfectly. One should not, therefore, select a strategy based on a deterministic analysis alone but perform sensitivity analyses as well before choosing any strategy.

This paper discusses the robustness of the GP, MM and CM strategies under uncertainty through sensitivity analyses. Specifically, we define the robustness of a network design strategy with two aspects: *financial* and *social*. *From the financial point of view, a robust strategy can maintain profitability or financial viability under a relatively large range of uncertainty. From the social point of view, a robust strategy can produce the highest total discounted consumer surplus (TDCS) among other alternatives under uncertainty.* The two major uncertainties considered here are the value of time and potential demands. The outline of this paper is as follows. Section 2 formulates the N-period traffic assignment as a variational inequality problem. Section 3 depicts the considerations involved in network design. Section 4 describes the three government strategies: the GP, MM and CM strategies. Section 5 presents the sensitivity analysis. Section 6 contains the numerical study to illustrate the formulation and performance of the three strategies under uncertainty. Finally, Section 7 provides some concluding remarks.

2. N-PERIOD TRAFFIC ASSIGNMENT

For the sake of completeness, this section provides the formulation for the N-period traffic assignment, which is equivalent to the lower level formulation of the network design problem in Lo and Szeto (2004c). We consider a general transportation network with multiple Origin-Destination (OD) flows over the planning horizon $[0,T]$. The horizon is divided into *N* equal planning intervals. Other notations adopted in this paper are summarized in the Appendix.

The N-period traffic assignment includes N time-dependent traffic assignment sub-problems. For each sub-problem, travelers are assumed to follow the Wardrop's principle (1952). This principle requires that path *p* between OD pair *rs* will not be used if its travel cost is higher than the minimum travel cost between OD pair *rs*. Conversely, any used path *p* must have its travel cost equal to the minimum travel cost between OD pair *rs*. Mathematically, this principle for each sub-problem in period τ can be expressed as:

$$f_{p,\tau}^{rs}\left(\eta_{p,\tau}^{rs}-\pi_{\tau}^{rs}\right)=0, \forall rs, p, \tau , \qquad (1)$$

$$\eta_{p,\tau}^{rs}-\pi_{\tau}^{rs}\geq 0, \forall rs, p, \tau , \qquad (2)$$

where $f_{p,\tau}^{rs}$ and $\eta_{p,\tau}^{rs}$ are respectively the representative hourly flow and the path travel cost for path *p* between OD pair *rs* in period τ, and; π_{τ}^{rs} is the lowest travel cost.

Equations (1) and (2) constitute the nonlinear complementarity conditions for the traffic assignment principle for each period τ. According to (1), if path *p* carries a positive flow in period τ, (i.e., $f_{p,\tau}^{rs} > 0$), then its associated path cost $\eta_{p,\tau}^{rs}$ must be equal to the lowest cost π_{τ}^{rs} through the condition $\left(\eta_{p,\tau}^{rs} - \pi_{\tau}^{rs}\right)$. Equation (2) ensures π_{τ}^{rs} to be the lowest cost among all the possible paths between OD pair *rs* in period τ.

The path travel cost $\eta_{p,\tau}^{rs}$ and the minimum travel cost π_{τ}^{rs} can be determined once the equilibrium path flows **f** are known, written as:

$$v_{a,\tau} = \sum_{rs}\sum_{p} f_{p,\tau}^{rs}\delta_a^p, \forall a, \tau, \tag{3}$$

$$t_{a,\tau} = t_a^0\left(1 + \overline{\alpha}\left(\frac{v_{a,\tau}}{c_a + \sum_{i=1}^{\tau} y_{a,i}}\right)^{\overline{\beta}}\right), \forall a, \tau, \tag{4}$$

$$\eta_{p,\tau}^{rs} = \sum_{a \in A/B} \psi t_{a,\tau}\delta_a^p + \sum_{b \in B}\left(\psi t_{b,\tau} + \rho_{b,\tau}\right)\delta_b^p, \forall rs, p, \tau, \tag{5}$$

where

$v_{a,\tau}$ is the representative hourly flow on link *a* in period τ;

$t_{a,\tau}$ is the travel time on link *a* in period τ;

$y_{a,\tau}$ is the capacity enhancement on link *a* in period τ, meaning that the capacity of link *a* is increased by $y_{a,\tau}$ units at the beginning of period τ;

δ_a^p is a link-path incidence indicator, which equals one if link *a* is on path *p*, zero otherwise;

t_a^0 is the free-flow travel time of link *a*;

c_a is the capacity of link *a* before the capacity improvement;

$\overline{\alpha}, \overline{\beta}$ are parameters of the link performance function;

ψ is the cost of unit travel time;

$\rho_{b,\tau}$ is the toll on toll link *b* in period τ;

A is a set of links, and;

B is a set of toll links, $B \subset A$.

Equation (3) states that link flow is obtained by summing the corresponding path flows on the link. Equation (4) is a typical link performance function. The summation term in (4) represents the total capacity enhancements of link *a* up to period τ. Therefore, the denominator inside the bracket denotes the link capacity in period τ after implementing the enhancements before and inclusive of period τ. When $\overline{\alpha} = 0.15$ and $\overline{\beta} = 4$, equation (4) is reduced to the typical Bureau of Public Roads (BPR) function. Equation (5) computes the path travel cost based on the corresponding link travel costs. The first term is the sum of the travel-time costs $\psi t_{a,\tau}$ on toll-free links on the path whereas the second term is the sum of the travel costs on toll links on the path in which the travel cost on each toll link is the sum of the travel-time cost $\psi t_{b,\tau}$ and toll $\rho_{b,\tau}$ on that link.

Each traffic assignment sub-problem also includes the flow conservation and non-negativity conditions, expressed as:

$$\sum_{p} f_{p,\tau}^{rs} = q_{\tau}^{rs}, \forall rs, \tau , \tag{6}$$

$$f_{p,\tau}^{rs} \geq 0, \forall rs, p, \tau , \tag{7}$$

where q_{τ}^{rs} is the travel demand of OD pair *rs* in period τ.

The travel demand of OD pair *rs* in period τ, q_{τ}^{rs}, is modeled as an elastic function of the potential demand $\tilde{q}_{\tau}^{rs}$ and its lowest travel cost π_{τ}^{rs} of that period. This elastic travel demand function is generally decreasing, implying that higher travel costs lead to lower demands. Many functional forms can be adopted in this framework without difficulty. For the purpose of illustration, in this study, the following travel demand function is adopted:

$$q_{\tau}^{rs} = \tilde{q}_{\tau}^{rs} - \gamma^{rs} \pi_{\tau}^{rs} , \tag{8}$$

where γ^{rs} is the parameter of the travel demand function of OD pair *rs*.

The potential demand per OD pair of each period represents the potential travel growth due to population growth, and is modeled to depend on the potential demand of the last period but not to depend on the traffic conditions. For simplicity, in this paper, the potential demand function is defined as:

$$\tilde{q}_{\tau}^{rs} = \tilde{q}_{\tau-1}^{rs} \left(1 + h^{rs}\right), \tag{9}$$

where h^{rs} is the growth rate of potential demand between OD pair *rs*.

The N-period traffic assignment is to find $\mathbf{f} = \left(f_{p,\tau}^{rs}\right)$ to satisfy (1)-(9), which can be formulated as a path-based variational inequality problem (VIP). To derive this VIP, we first derive the path-based nonlinear complementarity problem (NCP) from the N-period traffic assignment problem (1)-(9). The NCP is to find an optimal vector $\mathbf{u}^* \geq \mathbf{0}$ to satisfy

$$\mathbf{H}\left(\mathbf{u}^*\right) \geq \mathbf{0} \text{ and } \mathbf{u}^{*T} \mathbf{H}\left(\mathbf{u}^*\right) = 0 , \tag{10}$$

where **H** is the mapping function of the variable **u**, and the asterisk associated with the variable refers to the optimal solution.

According to conditions (1), (2), and (7), these conditions are in a NCP format with

$$\mathbf{u} = \mathbf{f} \text{ , and} \tag{11}$$

$$\mathbf{H}(\mathbf{u}) = \mathbf{H}(\mathbf{f}) = \left(\eta_{p,\tau}^{rs} - \pi_{\tau}^{rs}\right). \tag{12}$$

Equation (12) implies that the mapping function **H** is a function the decision variable **f**. From (6), (8), and (9), the lowest travel cost π_{τ}^{rs} can be written as:

$$\pi_{\tau}^{rs} = \frac{\tilde{q}_1^{rs} \left(1 + h^{rs}\right)^{\tau-1} - \sum_p f_{p,\tau}^{rs}}{\gamma^{rs}} . \tag{13}$$

With $\eta_{p,\tau}^{rs}$ defined by (3)-(5) and π_{τ}^{rs} defined by (13), the N-period traffic assignment problem can be expressed as the NCP (10)-(12).

The path-based NCP (10)-(12) can be transformed to a path-based variational inequality problem (VIP) (Proposition 1.4 in Nagurney, 1999): to find $\mathbf{f}^* \in \Omega$ such that

$$\mathbf{H}\left(\mathbf{f}^*\right)^T\left(\mathbf{f}-\mathbf{f}^*\right) \geq 0, \forall \mathbf{f} \in \Omega, \tag{14}$$

where Ω is the solution set of the N-period traffic assignment problem. According to this proposition, both the VIP (14) and the NCP (10)-(12) have the same mapping function and solution set.

3. CONSIDERATIONS IN NETWORK DESIGN

In general, transport infrastructures are built by either the private sector or the government. These two parties have different objectives: (i) each private toll road operator is concerned with its own *total discounted profit*; and (ii) the government is primarily concerned with the *total discounted consumer surplus* and, to a lesser extent, with the *total discounted social surplus*. This section describes their considerations in network design.

3.1 Total discounted profit

The total discounted profit on link *b*, P_b, is the sum of the total discounted toll revenue from toll link *b* minus the sum of the corresponding total discounted improvement cost. Mathematically, it can be stated as:

$$P_b = \sum_{\tau} P_{b,\tau} = \sum\left(R_{b,\tau} - C_{b,\tau}\right), \tag{15}$$

where $P_{b,\tau}$ is the discounted profit from link *b* in period τ, $R_{b,\tau}$ is the discounted revenue from link *b* in period τ, and $C_{b,\tau}$ is the discounted improvement cost from link *b* in period τ.

The discounted toll revenue $R_{b,\tau}$ collected from link *b* in period τ in (15) is defined as the product of the toll $\rho_{b,\tau}$ and the volume $nv_{b,\tau}$ on that link in period τ, discounted to present value terms. Mathematically, the discounted toll revenue $R_{b,\tau}$ is written as:

$$R_{b,\tau} = \frac{\rho_{b,\tau}\left(nv_{b,\tau}\right)}{\left(1+i\right)^{\tau-1}}, \tag{16}$$

where $1/\left(1+i\right)^{\tau-1}$ is the discount factor for period τ, *i* is the interest rate, and *n* is a factor converting the link volume $v_{b,\tau}$ from an hourly basis to a period basis.

The discounted improvement cost $C_{b,\tau}$ in (15) is expressed as:

$$C_{b,\tau} = \frac{g_b\left(v_{b,\tau}, \tau\right)}{\left(1+i\right)^{\tau-1}}, \tag{17}$$

$$g_b\left(v_{b,\tau}, \tau\right) = \mu t_b^0 y_{b,\tau}\left(1+r\right)^{\tau-1}, \tag{18}$$

where $g_b(.)$ is an improvement cost function, and μ is a construction cost parameter.

The term $1/(1+r)^{\tau-1}$ in (18) represents the inflation factor: for the same capacity enhancement, the improvement cost increases by *r*% each period. The term $\mu t_b^0 y_{b,1}$ models the improvement cost of link *b* in the base period. Equation (18) depicts the general relationship that the improvement cost of a link is proportional to the extent of the widening (and hence the capacity gain $y_{b,\tau}$) and its length (as represented by its free-flow travel time t_b^0). Moreover, the higher the construction cost parameter, the higher the construction cost. The function (18) is adopted for illustration and simplicity; other functional forms can be adopted in this framework without difficulty.

The total discounted profit (TDP) is the sum of the total discounted profit of each toll link, which can be formulated as:

$$\text{TDP} = \sum_b P_b \,. \tag{19}$$

3.2 Total discounted consumer surplus

The total discounted consumer surplus (TDCS) relates to consumer surplus (CS), which measures the difference between what consumers would be willing to pay for travel and what they actually pay. It internalizes the effect of network congestion and the public's propensity to travel. For the same network and demand characteristics, a higher CS implies a lower travel cost and a better performing system. Mathematically, the CS of OD pair *rs* in period τ is expressed as:

$$\text{CS}_\tau^{rs} = n\left(\int_0^{q_\tau^{rs}} D_\tau^{rs-1}(v)\,\mathrm{d}v - \pi_\tau^{rs} q_\tau^{rs}\right), \tag{20}$$

where *n* is a factor converting CS from an hourly basis to a period basis, and $D_\tau^{rs-1}(.)$ is the inverse demand function for OD pair *rs* in period τ.

The first term in the square bracket in (20) is the total travel cost the demand q_τ^{rs} would be willing to pay whereas the second term is the total travel cost they actually pay. TDCS is then obtained by summing the consumer surplus measure, adjusted to present value, over the planning horizon for all OD pairs, representing the overall network discounted consumer surplus over the planning horizon. The total consumer surplus, discounted to present value terms, is written as:

$$\text{TDCS} = \sum_\tau \sum_{rs} \frac{\text{CS}_\tau^{rs}}{(1+i)^{\tau-1}} \,. \tag{21}$$

3.3 Total discounted social surplus

The total discounted social surplus (TDSS) is defined as the sum of the total discounted profit (TDP) and the total discounted consumer surplus (TDCS), formulated as:

$$\text{TDSS} = \text{TDP} + \text{TDCS}. \tag{22}$$

According to (22), when TDP is zero, TDSS is equal to TDCS.

4. THREE GOVERNMENT STRATEGIES

The government has three strategies, among others, to improve the transportation network over the planning horizon without relying on public expenditures to finance the improvements. The three strategies are the government-as-the-provider (GP) strategy, the monopoly market (MM) strategy, and the competitive market (CM) strategy. These strategies will be depicted in this section and are summarized in Table 1.

TABLE 1: A comparison of three government strategies

Strategies	Builder(s) and operator(s)	Operation and management methods	Objective
Government-as-the-provider	Government	Cost recovery or zero profit	Maximize TDCS or TDSS
Monopoly market	1 private entity	Build-Operate-Transfer (BOT)	Maximize the total discounted profit (TDP)
Competitive market	Multiple private entities	Competitive BOT	Maximize the individual discounted profit

4.1 The government-as-the-provider strategy

In the GP strategy, the government provides the infrastructure improvements and collects just enough tolls to recover the costs. Since the builder and operator of the toll roads are both the government, the tolling and network improvement strategy follow the government perspective, in which the primary consideration is on the network performance, as measured by TDCS. Thus, the objective of the GP strategy is to design the capacity improvements and tolls so as to maximize TDCS based on the principle of exact cost recovery. Equivalently, the objective of the GP strategy is to design the capacity improvements and tolls so as to maximize TDSS subject to the zero profit condition.

4.2 The monopoly market strategy

In this MM strategy, the government allows one private entity to build all the toll roads. The private entity will then collect tolls on these toll roads within a franchised period; and after the franchised period is over, all these toll roads are transferred back to the government. In other words, the government adopts the Build-Operate-Transfer (BOT) scheme for network improvements. The tolling and improvement strategies follow the profit maximization perspective of the private entity. Consequently, the objective of this strategy is to design the improvements and tolls so as to maximize the total discounted profit (TDP).

4.3 The competitive market strategy

In the CM strategy, the government allows multiple operators to build different toll roads and collect tolls in a competitive manner. The operation and management of each toll road follow the BOT scheme. Under this CM strategy, each operator is concerned with its individual profit, which depends on both its as well as others' tolling and improvement strategies. We adopt the Nash equilibrium to represent this strategy.

5. OUTLINE OF SENSITIVITY ANALYSIS

To understand the effects of uncertainty on the three strategies, we conduct a sensitivity analysis through simulation of two important parameters, namely the value of time and potential demands. The methodology is as follows: Using the optimal tolls and capacity improvements obtained under the three strategies, we repeatedly solve the N-period traffic assignment expressed as the VIP (14) under a range of parameter values and determine the corresponding set of total consumer surplus, profit, and social surplus, discounted to the present value, based on equations (20)-(22). We then examine the effect of each parameter considered on TDP, TDCS, and TDSS by plotting them against the parameter values considered. The procedure for each design strategy is summarized as follows:

(i) Determine the optimal tolls and improvements based on the predicted parameter values using the methodology described in Lo and Szeto (2004c).
(ii) Set the parameter values.
(iii) Solve the N-period traffic assignment formulated as the VIP (14).
(iv) Determine TDP, TDCS, and TDSS based on equations (20)-(22).
(v) Repeat steps (ii)-(iv) until all possible values of the parameter(s) are selected.
(vi) Plot TDP, TDCS, and TDSS against the parameter values.

To solve the VIP (14) in step (iii), we choose the projection method developed by Han and Lo (2004). The outline of this method is as follows:

(i) Select positive constants[3] $\bar{\gamma}$ and $\bar{t}$ such that $\bar{\gamma} < 4\bar{\psi}$, $\bar{t} = \delta\left(1 - \frac{\bar{\gamma}}{4\bar{\psi}}\right)$, $\delta \in (0,2)$, $\bar{t} \in (0,1)$, where $\bar{\psi}$ is the modulus constant associated with $\mathbf{H}(\mathbf{f})$.
(ii) Start with an initial point $\mathbf{x}^0 \in \Omega$.
(iii) Generate $\mathbf{f}^{k+1} = \mathbf{f}^k - \bar{t} \cdot \mathbf{e}(\mathbf{f}^k, \bar{\gamma})$, where $\mathbf{e}(\mathbf{f}^k, \bar{\gamma}) = \mathbf{f} - P_\Omega\{\mathbf{f} - \bar{\gamma} \cdot \mathbf{H}(\mathbf{f})\}$, and $P_\Omega\{.\}$ is the projection on the solution set Ω.
(iv) Convergence check, let ε be the convergence criterion: If $\left\|\mathbf{e}(\mathbf{f}^k, \bar{\gamma})\right\|^2 \le \varepsilon$ stop, otherwise $k = k + 1$, go to (iii).

6. NUMERICAL STUDY

For comparison purposes, the sensitivity analysis is based on the scenario as in Lo and Szeto (2004c). The parameters considered include the value of time and potential demands. Specifically, this section focuses on the following questions:

1. Would the GP strategy maintain full or exact cost recovery under uncertainty?
2. Would the MM and CM strategies maintain profitability under uncertainty?
3. Would the GP strategy produce the largest TDCS among other alternatives under uncertainty?
4. Would the MM and CM strategies produce a similar level of TDCS as that of the GP strategy under uncertainty?

In the following, we include the scenario setting as well as the optimal tolls and improvements under perfect predictions for each design strategy.

---

[3] The selection involves some sort of trial and error initially. This procedure is generally necessary for most projection-based algorithms for VIP.

6.1 Scenario setting

Due to increases in demand, the government plans to build two toll roads in a city using one of the three strategies: the GP, MM, and CM strategies. The network, as shown in Figure 1, consists of four nodes, six links, and two OD pairs. The two OD pairs are from node 1 to node 2 and from node 4 to node 3. Links 1 and 3 are the proposed toll roads which are represented by dashed lines. The parameters in this scenario include:

(a) Initial link capacities: $c_2 = c_4 = 3600$ vph , $c_5 = c_6 = 1800$ vph
(b) Free-flow travel times: t_1^0 = 12 min, $t_2^0 = t_3^0 = t_4^0$ = 5 min, t_5^0 = 25 min, t_6^0 = 15 min
(c) Potential demands at period 1: $\tilde{q}_1^{12}$ = 9000 vph, $\tilde{q}_1^{42}$ = 7000 vph
(d) Growth rates: h^{12} = 0.04, h^{42} = 0.02
(e) Link performance function parameters: $\overline{\alpha}$ = 0.15, $\overline{\beta}$ = 4
(f) Parameters of the travel demand functions: $\gamma^{12} = \gamma^{42}$ = 100 veh/h^2
(g) Interest and inflation rates: i = 0.03, r = 0.01
(h) Value of time: ψ = HK\$30/h
(i) Converting factor: n = 87600
(j) Construction cost parameter: μ = HK\$6000000/veh
(k) Length of each period: 10 years
(l) Planning horizon and franchised period: [0, 30]

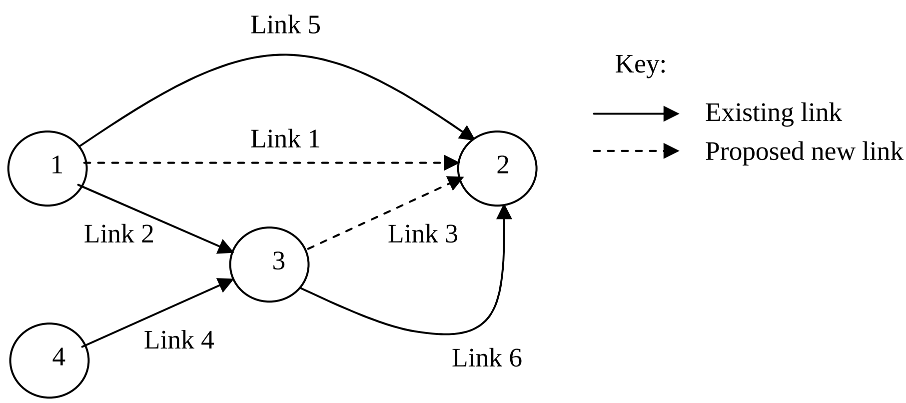

FIGURE 1: The example network

6.2 Optimal solutions assuming perfect predictions

Table 2 and Figure 2, respectively, show the optimal capacity enhancements and tolls over time for the three strategies assuming no uncertainty. The strategies produce markedly different toll and improvement schemes. The GP strategy introduces low toll charges and large capacity improvements. The MM strategy, on the other hand, introduces high toll charges and low capacity improvements. The CM strategy adopts toll and improvement levels that are somewhat between the GP and MM strategies. In terms of toll charges, those of the MM and CM strategies increase over time, whereas that of the GP strategy decreases over time. Finally, the MM strategy only improves link

3 whereas the GP and CM strategies improve both links 1 and 3. These tolls and capacities will be used in the sensitivity analysis in the following subsections.

TABLE 2: Capacity improvements over time for the three strategies

Period	The GP strategy	The CM strategy	The MM strategy
(a) Capacity improvements on link 1			
1	4061.3	3356.3	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0
(b) Capacity improvements on link 3			
Period	The GP strategy	The CM strategy	The MM strategy
1	9527.9	6721.9	4847.3
2	0.0	0.0	0.0
3	0.0	0.0	0.0

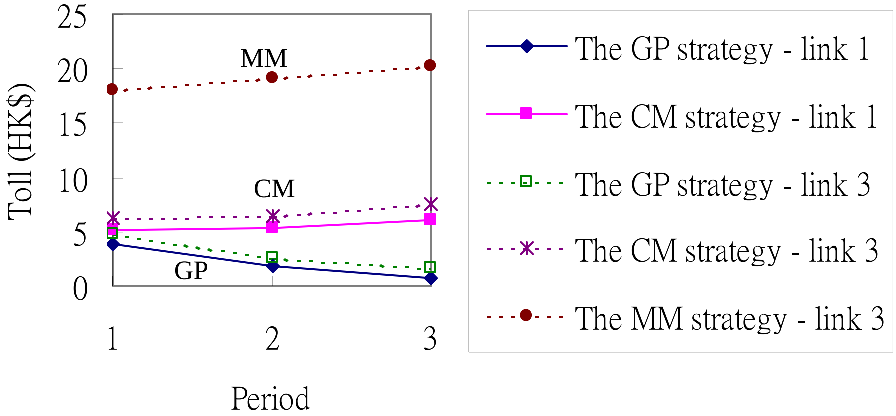

FIGURE 2: Tolls over time for the three strategies

6.3 Sensitivity analysis

6.3.1 Value of time

Figure 3 shows the total discounted profits from link 1, from link 3, and their sum over the planning horizon under a range of values of time for the GP strategy. The x-axis represents the value of time whereas the y-axis represents the total discounted profit. The top curve is for the total discounted profit from link 3; the bottom curve is for the total discounted profit from link 1; the middle curve is obtained by adding the top and bottom curves. According to the top curve, the total discounted profit from link 3 increases when the value of time ψ increases from HK$ 1 to HK$ 31, and decreases with the value of time in the range from HK$ 31 to HK$ 60. Moreover, the total discounted profit from link 3 is positive only when the value of time is greater than HK$ 11, implying link 3 can subsidize link 1 only when the value of time is greater than HK$ 11; otherwise, link 3 itself also requires subsidy. There are some non-differentiable points on this curve, for example at ψ = HK$12, 21, 31, due to changes in the used path set across the range of values of time.

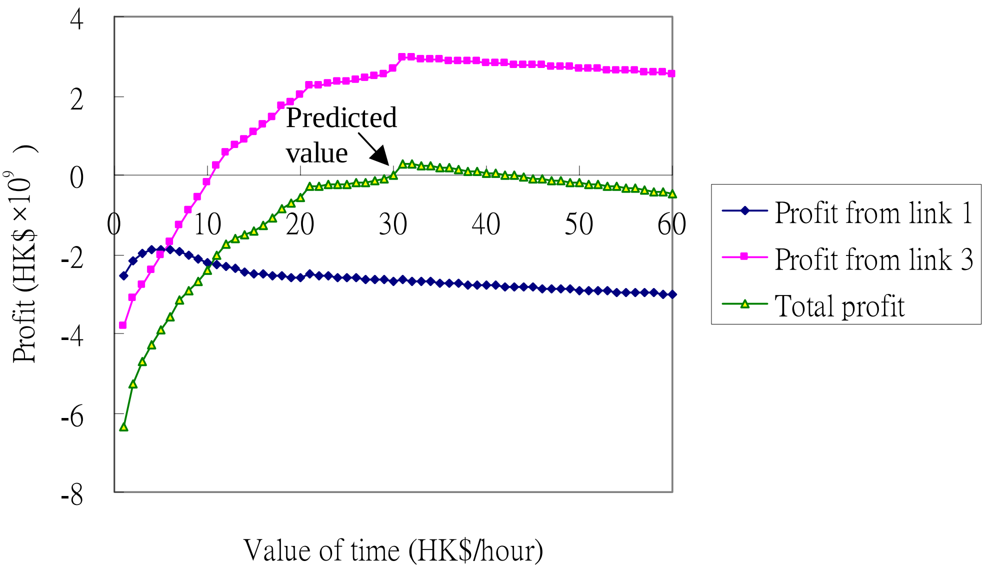

FIGURE 3: Profits under a range of values of time for the GP strategy

According to the bottom curve, the total discounted profit from link 1 is always negative meaning that this link always requires subsidies. The end result is that the total discounted profit (the middle curve) is non-negative only when the value of time is between HK$30 (the predicted value) to HK$42, in which the profit from link 3 is high enough to subsidize link 1; full or exact cost recovery is possible within this range. This finding implies that *the GP strategy can only achieve cost recovery within a very narrow range of values of time.*

Figure 4 shows the discounted profits under a range of values of time for the MM and CM strategies. The x-axis represents the value of time whereas the y-axis represents the discounted profit. The legend is shown on the right hand side of the figure, in which Operator 1 (3) refers to the toll road operator for link 1 (3). The upper curve is for the MM strategy; the lower two curves are for the CM strategy. For the upper curve, the total discounted profit of the monopoly increases with the value of time monotonically up to about HK$$25\times10^9$. The result illustrates that the monopoly can maintain profitability when the value of time is greater than HK$3. For the middle curve, the total discounted profit of Operator 1 increases monotonically with the value of time, and is positive when the value of time is greater than HK$4. For the lower curve, the total discounted profit of Operator 3 increases with the value of time in the range from HK$1 to HK$5, and then decreases outside this range. In addition, the total discounted profit is positive in the whole range of values of time considered, implying that Operator 3 can maintain profitability under the range of values of time considered. Based on these findings, we can conclude that *though not always possible, the MM and CM strategies can maintain profitability under a relatively wider range of values of time.*

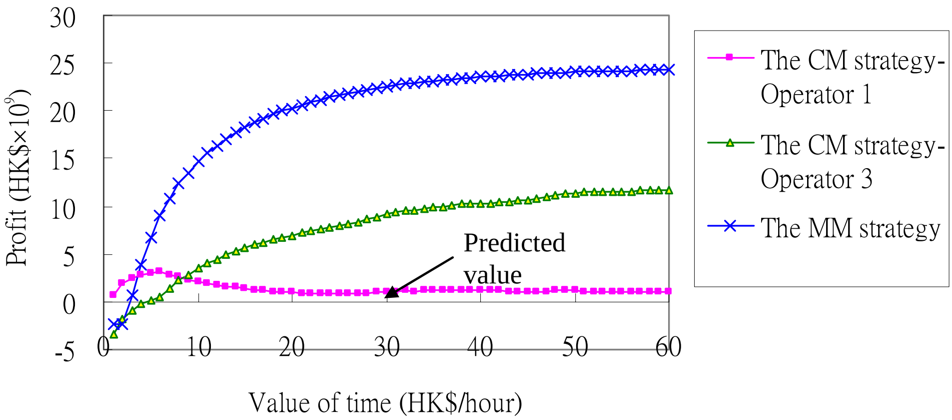

FIGURE 4: Profits under a range of values of time for the MM and CM strategies

Figure 5 and Figure 6 show, respectively, the total discounted consumer surpluses (TDCS) and total discounted social surpluses (TDSS) under a range of values of time for the three strategies and the do-nothing strategy. As revealed by Figure 5, in general, as the value of time increases, TDCS decreases in each strategy, because a high value of time implies a high travel cost, which in turn implies a low travel demand and consumer surplus. In particular, for a fixed value of time, the TDCS produced by the GP strategy is larger than that of the CM strategy, which in turn is larger than that of the MM strategy. Comparing with the do-nothing strategy, all three strategies are better for a wide range of values of time. These findings indicate that the GP strategy produces the best TDCS, and that all three strategies are better than the do-nothing strategy. One surprising result is that the CM strategy produces a TDCS that is close to that of the GP strategy whereas the MM strategy does not. It appears that the competition does help to keep the balance between making profits and introducing substantial network improvements. In terms of TDSS, similar results are observed in Figure 6.

There is a tradeoff between maintaining profitability and maximizing TDCS or TDSS under uncertain values of time. Although the GP strategy produces the best TDCS and TDSS under uncertain values of time, the government must shoulder the financial risk of not able to fully recover the costs. In particular, if the actual value of time were higher or lower than predicted, cost recovery would not be possible. The GP strategy is thus robust in its social aspect but not its financial aspect. The MM strategy can maintain profitability under a relatively wide range of values of time but produces a much lower TDCS and TDSS than the GP strategy. The MM strategy is therefore robust in its financial aspect but not its social aspect. It appears that the CM strategy can maintain profitability under a relatively wide range of values of time and produce TDCS and TDSS comparable to those of the GP strategy. The CM strategy seems to be a good strategy in coping with uncertain values of time.

The results are reasonable. Recall the results in Section 6.2 that under the ideal situation, the GP strategy introduces low toll charges and large capacity improvements whereas the MM strategy does the reverse; the CM strategy introduces the tolls and improvements somewhat in between the GP and MM strategies. As the GP strategy introduces the largest network improvements and the lowest toll revenue, the TDCS must be the largest. However, when the actual value of time is lower or higher than predicted, the actual demand would be lower, resulting in the risk of not able to fully recover the costs. On the other hand, the MM strategy collects high toll charges but introduces small network improvements. This strategy thus leaves a relatively large cushion for maintaining profitability. Likewise, the CM strategy does not spend all the toll revenue on network improvements, which help to maintain profitability. The competition, on the other hand, helps to drive down the toll charges, which in turn leads to a larger TDCS compared with that the MM strategy.

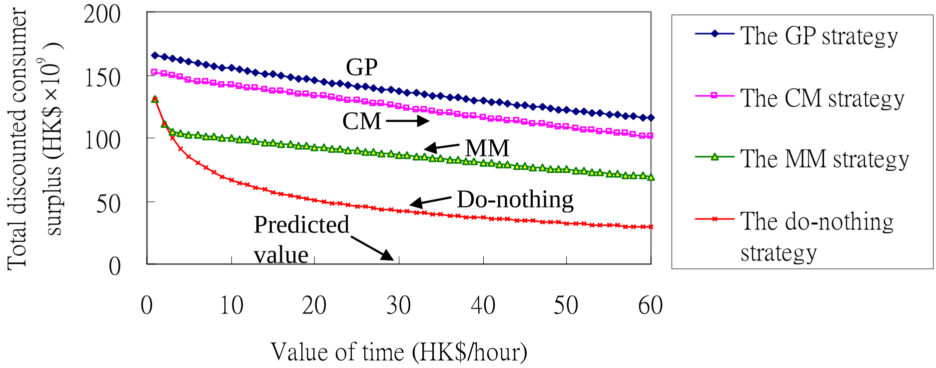

FIGURE 5: Total discounted consumer surpluses under a range of values of time

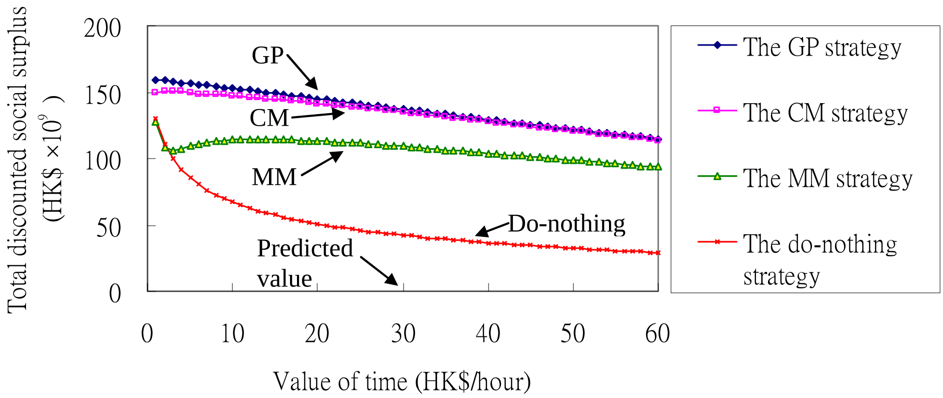

FIGURE 6: Total discounted social surpluses under a range of values of time

6.3.2 Potential demands

Figure 7 plots the discounted profit for each strategy under a range of potential demands in the reference time period (the first period). The x-axis is the ratio of the actual potential demand to the predicted one for each OD pair. The y-axis is the discounted profit. The top, middle two, and bottom curves are for the GP, CM and MM strategies respectively. In general, the profit increases with the ratio: the higher is the actual potential demand, the higher is the discounted profit. Specifically, the discounted profit under the GP strategy is positive when the ratio is larger than one, zero when the ratio is one, and negative otherwise. This means that *the GP strategy can only maintain cost recovery when the actual potential demands are equal to or greater than what are* *predicted*. The MM and CM strategies produce the discounted profits under a wider range of ratios when compared with the GP strategy. The MM and CM strategies make profits when the ratios are larger than or equal to 0.8 and 0.9 respectively. *The MM and CM strategies can maintain profits under a relatively wider range of potential demands.*

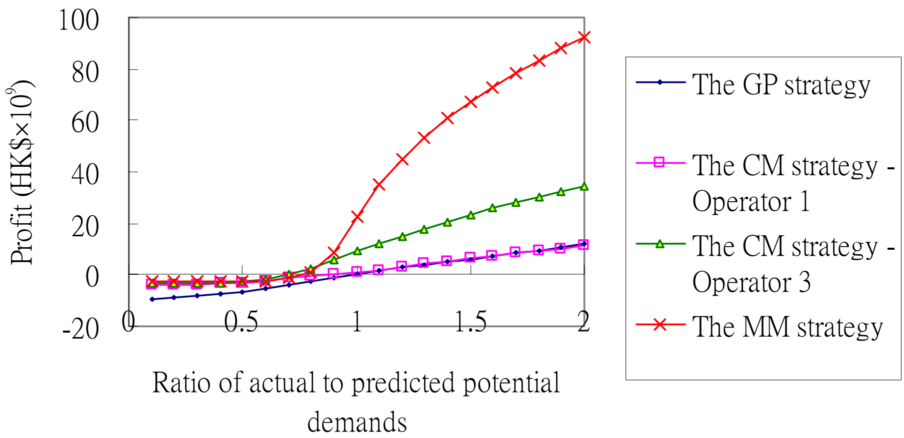

FIGURE 7: Profits under a range of potential demands

Figure 8 shows the TDCS under each design strategy. The x-axis is the ratio of the actual potential demand to the predicted one for each OD pair whereas the y-axis is TDCS. Generally, TDCS increases with the ratio. Moreover, the GP strategy produces the highest TDCS. In particular, the performance of the CM strategy is close to that of the GP strategy in terms of TDCS.

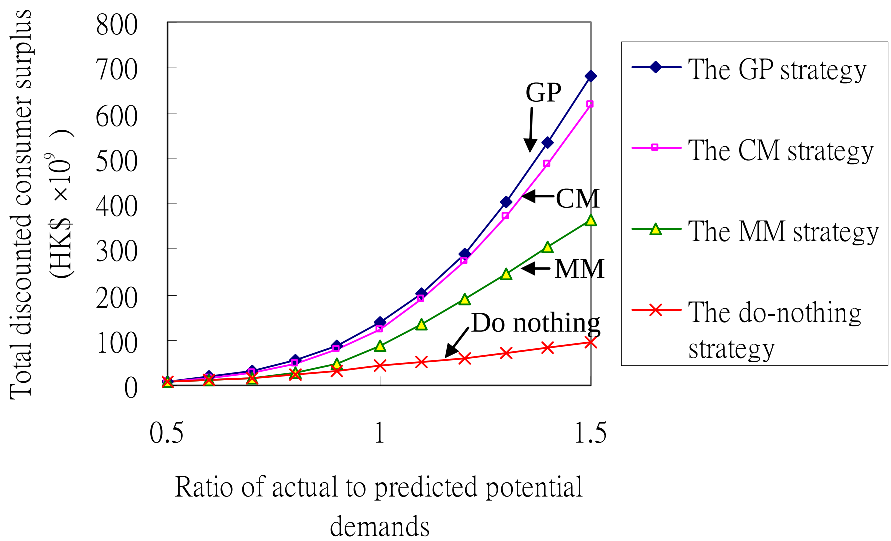

FIGURE 8: Total discounted consumer surpluses under a range of potential demands

7. CONCLUDING REMARKS

This paper discusses the social and financial aspects of three possible government's network improvement strategies, namely government-as-the-provider (GP), monopoly market (MM), and competitive market (CM), under uncertain demand and value of time. Numerical results of a small network with multiple OD flows are provided to illustrate these two aspects. Although the results are likely to be network specific and scenario dependent, they are quite revealing. The results show that in general, in selecting a network improvement strategy, the government faces the tradeoff between maintaining financial viability (financial aspect) and maximizing TDCS (social aspect) under uncertainty.

The GP strategy would most likely produce the largest TDCS but maintaining cost recovery could be a problem. On the other hand, the MM strategy would most likely be profitable to the operators, at the expense of resulting in the lowest TDCS among the three strategies. The CM strategy seems to be able to maintain a balance between TDCS and profitability under uncertainty. This finding is consistent with the current trend of privatizing government services to multiple companies. However, the CM strategy is not a panacea for all situations. There are situations wherein competition could be wasteful, especially when the overall demand is low, rendering each competitive operator not having enough revenue to sustain its operations. Should this be the case, the model developed by this study ought to be able to pinpoint the problem and advise against such a development. On the other hand, competition could also bring about other undesirable outcomes, such as in the form of collusion. In that case, the government must monitor the competition and introduce regulations or policies if necessary to properly constrain it.

We believe the analysis and discussions here have brought up a number of research extensions. Firstly, whether there must be a tradeoff between the financial and social aspects is to be further investigated. Secondly, network improvements are often subject to economies of scale and the associated maintenance costs. These two factors should be duly considered for a complete analysis. Thirdly, the network improvements will produce benefits beyond the planning horizon. Nevertheless, the analysis here did not include such benefits. One possible future research direction is to capture these benefits in the analysis. Finally, land use changes are not included in the formulation. However, using the formulations developed in Lo and Szeto (2004c) as a platform, one can extend the consideration to include these issues.

ACKNOWLEDGEMENTS

This research is sponsored by the Competitive Earmarked Research Grant HKUST6154/03E from the Hong Kong Research Grant Council. We are grateful for the constructive comments of the two referees.

REFERENCES

Boyce, D.E. and Janson, B.N. (1980) A discrete transportation network design problem with combined trip distribution and assignment. *Transportation Research*, **14B**, 147-154.

Chen, M.Y. and Alfa, A.S. (1991) A network design algorithm using a stochastic incremental traffic assignment approach. *Transportation Science*, **25**, 215-224.

Davis, G.A. (1994) Exact local solution of the continuous network design problem via stochastic user equilibrium assignment. *Transportation Research*, **28B**, 61-75.

Friesz, T.L., Anandalingam, G., Mehta, N.J., Nam, K., Shah, S.J. and Tobin, R.L. (1993) The multiobjective equilibrium network design problem revisited: a simulated annealing approach, *European Journal of Operational Research*, **65**, 44-57.

Han, D. and Lo, H. (2004) Solving nonadditive traffic assignment problems: A decent method for co-coercive variational inequalities. *European Journal of Operational Research*, in press.

LeBlanc, L.J. (1975) An algorithm for discrete network design problem. *Transportation Science*, **9**, 183-199.

Lo, H. and Szeto, W.Y. (2003) Time-dependent transport network design: a study of budget sensitivity. *Journal of the Eastern Asia Society for Transportation Studies*, **5**, 1124-1139.

Lo, H. and Szeto, W.Y. (2004a) Planning transport network improvements over time. In Lee, D.H. (ed.), Chapter 9, *Urban and Regional Transportation Modeling*: *Essays in Honor of David Boyce*, Edward Elgar Publishing, pp. 157-176.

Lo, H. and Szeto, W.Y. (2004b) Time-dependent transport network design under cost-recovery. *Transportation Research B*, submitted.

Lo, H. and Szeto, W.Y. (2004c) Strategies for network improvements over time. *Proceeding of the 83rd Annual Meeting on Transportation Research Board*.

Marcotte, P. (1986) Network design problem with congestion effects: a case of bilevel programming. *Mathematical Programming*, **34,** 142-162.

Nagurney, A. (1999) *Network Economics: A Variational Inequality Approach*. Second Revised Edition. Norwell, Massachusetts, USA, Kluwer Academic Publishers.

Wardrop, J. (1952) Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers*, Part II, 325-378.

Yang, H. and Bell, M.G.H. (1998) Models and algorithms for road network design: a review and some new developments. *Transport Reviews*, **18**, 257-278.

Yang, H. and Meng, Q. (2000) Highway pricing and capacity choice in a road network under a build-operate-transfer scheme. *Transportation Research*, **34A**, 207-222.

APPENDIX

Set notations

A	set of links
B	set of toll links, $B \subset A$

Indices

rs	OD pair
p	path between OD pair rs
a	link, $a \in A$
b	toll link, $b \in B$
τ	period, $\tau \in \{1, \ldots, N\}$

Variables to be determined

$f_{p,\tau}^{rs}$	representative hourly flow on path *p* between OD pair *rs* in period τ
f	column vector of $\{f_{p,\tau}^{rs}\}$

Parameters given

i, *r*	discount and inflation rates
$\overline{\alpha}, \overline{\beta}$	parameters in the link performance function
μ	construction cost parameter in the construction cost function
ψ	cost of unit travel time.
γ^{rs}	parameter in the travel demand function
h^{rs}	growth rate of potential demand between OD pair *rs*
c_a	initial capacity of link *a*
t_a^0	free-flow travel time of link *a*
δ_a^p	link-path incidence indicator, $\delta_a^p = 1$ if *a* is on *p*, $\delta_a^p = 0$ otherwise
$\tilde{q}_1^{rs}$	potential demand between OD pair *rs* in the first period
$y_{a,\tau}$	capacity enhancement in period τ
$\rho_{a,\tau}$	toll on link *b* in period τ

Functions of capacity enhancements, tolls and/or equilibrium path flows

$v_{a,\tau}$	representative hourly flow on link *a* in period τ
$t_{a,\tau}$	travel time on link *a* in period τ
q_τ^{rs}	representative hourly travel demand between OD pair *rs* in period τ
$\eta_{p,\tau}^{rs}$	path travel cost for travelers taking path *p* between OD pair *rs* in period τ
π_τ^{rs}	the minimum travel cost between OD pair *rs* in period τ