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ADAPTATION OF THE PAIRED COMBINATORIAL LOGIT MODEL TO THE ROUTE CHOICE PROBLEM

SURACHET PRAVINVONGVUTH[1] AND ANTHONY CHEN[2]

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The paired combinatorial logit (PCL) model is one of the extended logit models recently developed from the generalized extreme value theory to handle the drawbacks of the multinomial logit (MNL) model. The PCL model is considered suitable for adaptation to the route choice problem to resolve the independence from irrelevant alternatives property inherited in the MNL model while keeping the analytical tractability of the logit choice probability function. In this paper, we adapt the PCL model to handle the two drawbacks of MNL in a route choice context: (1) inability to account for overlapping (or correlation) among routes, and (2) inability to account for perception variance with respect to trips of different lengths. Specifically, the effects of congestion, scaling, and overlapping on the PCL choice probabilities are examined using a set of carefully designed experiments and graphical illustrations. The experimental designs and graphical illustrations allow for intuitive explanation and comparison of the different effects on the calculation of the route choice probabilities.

KEYWORDS: Multinomial logit, paired combinatorial logit model, route choice

1. INTRODUCTION

Traffic assignment is an essential and fundamental step in the transportation planning process. It predicts the vehicular flows on the transportation network by assigning travel demands given in terms of an origin-destination trip table to routes in a network according to some route choice model. The User Equilibrium (UE) model is perhaps the most widely used route choice model in transportation planning practices. It is based on the well-known Wardrop's First Principle, which states:

> "*The journey times on all the routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route*" (Wardrop, 1952).

The criterion used in the route choice decision process is to minimize user travel time. All travelers are assumed to be rational, have perfect knowledge of network travel times, and capable of identifying the minimum travel time route. Given these assumptions, all travelers selfishly make their route choices that result in a stable equilibrium traffic flow pattern such that there is no incentive for anyone to change his/her route (Sheffi, 1985). The assumptions of perfect knowledge of network travel times and ability to always identify the minimum travel time route are rather unrealistic since travelers do not always have perfect perception of the network travel times, rarely do they know all possible routes in the network, and certainly do not always select the route just based on the minimum travel time criterion. To relax some of these assumptions, a random error term is incorporated in the route choice decision process to simulate travelers' imperfect perceptions of travel times such that they do not always end up picking the minimum travel time route. The random error term here is interpreted as the perception error of network travel times due to the travelers' imperfect knowledge of network conditions. In this model, each traveler is assumed to have some perceptions of the mean travel times on each link of the network, which include a random error term. Each traveler's route

[1] Department of Civil and Environmental Engineering, Utah State University, Logan, Utah, USA.
[2] Department of Civil and Environmental Engineering, Utah State University, Logan, Utah, USA. Corresponding author (E-mail: achen@cc.usu.edu).

choice criterion is to minimize the perceived value of the route travel time, which can be obtained by adding up the perceived travel times on all the links belonging to the route. This extension of the UE model is known as the stochastic user equilibrium (SUE) model and has the following definition.

"*At SUE, no motorists can improve his or her perceived travel time by unilaterally changing routes*" (Daganzo and Sheffi, 1977; Sheffi, 1985).

Route choice models proposed under this approach can have different specifications according modeling assumptions on the random error term. The two commonly used random error terms are Gumbel (Dial, 1971) and normal (Daganzo and Sheffi, 1977) distributions, which result in the logit- and probit-based route choice models. The multinomial logit (MNL) route choice model has a closed-form probability expression and an equivalent mathematical programming formulation (Fisk, 1980), and can be solved using both path enumeration techniques (Ben-Akiva et al., 1984; Cascetta et al., 1997; 2002) and column generation techniques (Bell et al., 1993; Bell, 1994; Chen and Alfa, 1991; Damberg et al., 1996; Huang and Bell, 1998; Leurent, 1997; Maher, 1998). The drawbacks of the MNL model are: (i) inability to account for overlapping (or correlation) among routes and (ii) inability to account for perception variance with respect to trips of different lengths. These two drawbacks stem from the logit's underlying assumptions that the random error terms are independently and identically Gumbel distributed (IID) with the same, fixed variances (Sheffi, 1985). Mathematically, the variance-covariance matrix of the MNL model is assumed to be a homoscedastic and diagonal matrix (Ben-Avika and Lerman, 1985). The multinomial probit (MNP) route choice model, on the other hand, does not have such drawbacks, because it handles the overlapping and identical variance problems between routes by allowing covariance between the random error terms for pairs of routes. However, the MNP model does not have a closed-form solution and it is computationally burdensome when the choice set contains more than a handful of routes. Due to the lack of a closed-form probability expression, solving the MNP route choice model typically requires the use of Monte Carlo simulation (Sheffi and Powell, 1982), Clark's approximation method (Maher and Hughes, 1997), or numerical integration methods (Rosa and Maher, 2002). Other specifications of the random error term include uniform (or rectangular) distribution (Burrell, 1968), triangular distribution (Tatineni, 1996; Tatineni et al., 1997), gamma distribution (Bovy, 1996; Bovy and Stern, 1990; Cantarella and Binetti, 2002; Sheffi, 1985), and lognormal distribution (Von Falkenshausen, 1966; Cantarella and Binetti, 1998). See Nielsen (1997) for a comparative study of the different distributions on the SUE traffic assignment models. It should be noted that the above specifications of the random error term only account for the randomness of the travelers' perceived travel times and treat the randomness of link travel times in the form of expected values. Mirchandani and Soroush (1987) were the first to propose the generalized traffic equilibrium model that incorporates both probabilistic travel times and variable perceptions in the route choice decision process. In their model, each traveler i is assumed to have a variable perception error that is normally distributed with a variable mean μ_i and a variable variance θ_i, where μ_i is assumed to be normally distributed over the population of travelers with zero mean and τ as the variance, and θ_i is assumed to be gamma distributed over the population of travelers with parameters α and β. This variable perception error allows each individual traveler to experience a different travel time for a given set of flows. See Chen and Recker (2001) and Chen and Ji (2002) for a review of four route choice models using network uncertainty and perception error as the classification factors.

Despite its theoretical drawbacks, the MNL model was widely used in many stochastic traffic assignment models due to its simple analytical form that allows easy implementation. Recently, there has been a renewed interest in using advanced discrete choice models derived from random utility maximization to improve the MNL model for the application of modeling the route choice problem. Several modifications of the logit structure have been proposed to relax the IID assumptions of the MNL model in the late 1990s. These extended logit models can be classified into three groups according to the model structures as follows.

(i) *Modifications of the MNL*: This group of models overcomes the overlapping problem by modifying the deterministic (or systematic) part of the utility to account for the overlapping problem while still retaining the single-level tree structure of the MNL model. The models in this group include the C-logit (Cascetta et al., 1996; Russo and Vitetta, 2003; Zhou and Chen, 2003), the implicit availability/perception (IAP) model (Cascetta et al., 2002), and the path-size logit (PSL) model (Ben-Avika and Bierlaire, 1999; Ramming, 2002). All three models add a correction term to the deterministic part of the utility to adjust the choice probability; however, the interpretation of each model is different. The C-logit model uses the commonality factor to penalize the overlapping paths, while both the IAP and PSL models use a logarithmic correction term to modify the utility (hence, the choice probability). In the IAP model, the logarithmic correction term accounts for the awareness of paths. If travelers are unaware of that path, or unable to use it, the logarithm correction term would decrease the probability of choosing path. On the other hand, the logarithmic correction term in the PSL model accounts for different path sizes determined by the length of links within a path and the relative lengths of paths that share a link. Zhou and Chen (2003) provided an equivalent mathematical programming formulation and a variational inequality formulation for the length-based and congestion-based C-logit models, respectively, while Russo and Vitetta (2003) provided a stochastic loading procedure for the length-based C-logit model.

(ii) *Generalized extreme value (GEV)*: The theory of GEV was proposed by McFadden (1978). However, not all GEV-type models are suitable for adaptation to the route choice problem (e.g., nested logit model) due to its complex tree structure and nesting requirements on the alternatives (Prashker and Behkor, 2004). Among the GEV-type models, cross nested logit (CNL) model (Prashker and Bekhor, 1998; Vovsha, and Bekhor, 1998; Papola, 2004), the generalized nested logit (GNL) model (Bekhor and Prashker, 2001), and the paired combinatorial logit (PCL) model (Bekhor and Prashker, 1999; Gliebe et al., 1999; Prashker and Bekhor, 1998; 2000) have been adapted to model the route choice problem. This group of GEV-type models captures the similarity among routes through the error component (instead of the deterministic component) of the utility function. The model structure in this group is a two-level tree structure, which allows alternative (route) to belong to more than one nest (i.e., a nest here is a link in the CNL and GNL models or a route pair in the PCL model). The choice probability is calculated according to the two-level tree structure using the marginal and conditional probabilities (see Figure 1 as an example of the PCL model). Equivalent mathematical programming formulations for all three models were given by Bekhor and Prashker (1999, 2001). A partial linearization algorithm for solving the PCL model was provided by Chen et al. (2003).

(iii) *Mixed logit model*: The mixed logit model, also known as logit kernel, random parameter logit, error component logit, and hybrid logit, is a highly flexible discrete choice model that is capable of approximating any random utility models (McFadden and Train, 2000). In addition, the mixed logit model overcomes three primary limitations of the standard MNL model by allowing for random taste variations, unrestricted substitute patterns, and correlation of unobserved factors over time (Train, 2002). Similar to the GEV-type model, the mixed logit model also captures the similarity among routes through the error component. However, the mixed logit model does not have a closed-form probability expression, because the error component is partitioned into two additive, uncorrelated terms: a probit-like term that captures the correlation among the alternatives and an IID Gumbel random variate. In general, the mixed logit model is defined on the basis of the functional form for its choice probabilities and can be defined under a variety of different behavioral specifications (Train, 2002). For the route choice problem, Bekhor et al. (2002) adopted the factor analytic logit kernel model to capture the similarity among routes. The downfall of the mixed logit model is that the probability expression does not exhibit generalized extreme values (i.e., no closed-form integral) and therefore, solving the choice probability would require either Monte Carlo simulation or numerical integration methods. Currently, there is no efficient path-based traffic assignment algorithm for solving the mixed logit route choice model.

For a more comprehensive review of the many extended logit models used in the route choice problem, please refer to Prashker and Bekhor (2004). In this paper, we focus on the adaptation of the PCL model to the route choice problem. This is because, among all the extended logit models, the PCL model is considered more suitable for adaptation to the route choice problem. It can resolve the drawbacks on the MNL model while keeping the analytical tractability of the choice probability function. In the PCL model, each pair of alternatives can have a similarity relationship that is completely independent of the similarity relationship of other pairs of alternatives. Bekhor and Prashker (1999) argued that this feature is highly desirable for modeling the route choice problem, since each pair of routes may have different similarities. Further, Gliebe et al. (1999) demonstrated using binomial and trinomial route choice examples without congestion effect to show that the PCL model could be scaled to account for perception variance with respect to different trip lengths. Both features are useful in addressing the well-known "independence from irrelevant alternatives" (IIA) property inherited in the MNL model. To investigate these features under congestion, we design a set of numerical experiments to examine how PCL handles the two drawbacks of MNL in a route choice context. Specifically, the effects of congestion, scaling, and overlapping on the PCL choice probabilities are examined using a set of carefully designed experiments and graphical illustrations. The experimental designs and graphical illustrations allow for intuitive explanation and comparison of the different effects on the calculation of the route choice probabilities.

2. THE PAIRED COMBINATORIAL LOGIT MODEL

The paired combinatorial logit (PCL) model was proposed by Chu (1989), further developed to demonstrate the structure, properties, and estimation by Koppleman and Wen (2000), and adapted to model the route choice problem by Bekhor and Prashker (1999) and Gliebe et al. (1999). The PCL model has a hierarchical structure that decomposes the choice probability into two levels represented by the marginal and conditional probabilities. Thus, the PCL choice probability can be expressed as:

$$P(k) = \sum_{j \neq k} P(kj) \cdot P(k / kj) , \tag{1}$$

where

$$P(k \mid kj) = \frac{e^{\frac{V_k}{1-\sigma_{kj}}}}{e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}}, \tag{2}$$

$$P(kj) = \frac{(1-\sigma_{kj})\left(e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}\right)^{1-\sigma_{kj}}}{\sum_{l=1}^{n-1}\sum_{m=l+1}^{n}(1-\sigma_{lm})\left(e^{\frac{V_l}{1-\sigma_{lm}}} + e^{\frac{V_m}{1-\sigma_{lm}}}\right)^{1-\sigma_{lm}}}, \tag{3}$$

V_k is the deterministic (observable) components of the utility for each alternative *k* (i.e., $U_k = V_k + \varepsilon_k, \forall k$), σ_{kj} is an index of the similarity between alternatives *k* and *j*, and *n* is the number of alternatives. $P(k \mid kj)$ is the conditional probability of choosing alternative *k* given that the alternative pair *kj* has been chosen, and $P(kj)$ is the marginal (unobserved) probability for the alternative pair *kj*. For a choice set of *n* alternatives, there are a total of *n*(*n*-1)/2 pairs of alternatives (as shown in the double summation of the denominator of the marginal probability). The PCL choice probability can also be derived from the GEV theory using the following generation function:

$$G(y_1, y_2, \ldots, y_n) = \sum_{k=1}^{n-1}\sum_{j=k+1}^{n}(1-\sigma_{kj})\left(y_k^{\frac{1}{1-\sigma_{kj}}} + y_j^{\frac{1}{1-\sigma_{kj}}}\right)^{1-\sigma_{kj}}, \tag{4}$$

where $y_k = e^{V_k}$. Then the choice probability of choosing alternative *k* is given as

$$P(k) = \frac{y_k G_k^{'}(y_1, y_2, \ldots, y_n)}{G(y_1, y_2, \ldots, y_n)} = \frac{\sum_{j \neq k}^{n} e^{\frac{V_k}{1-\sigma_{kj}}}(1-\sigma_{kj})\left(e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}\right)^{-\sigma_{kj}}}{\sum_{l=1}^{n-1}\sum_{m=l+1}^{n}(1-\sigma_{lm})\left(e^{\frac{V_l}{1-\sigma_{lm}}} + e^{\frac{V_m}{1-\sigma_{lm}}}\right)^{1-\sigma_{lm}}} \tag{5}$$

where $G_k^{'}(y_1, y_2, \ldots, y_n)$ is the derivative of the generation function with respect to y_k.

2.1 Comparison between MNL and PCL models for the route choice problem

To illustrate the differences between the two logit models, we use a route choice example given in Figure 1. The example network has three alternative routes between origin *r* and destination *s*. Route 2 overlaps with route 3 on link B, whereas route 1 is completely independent from the other two routes. Vovsha and Bekhor (1998) refer to this example network as the “blue route – red route” problem, analogous to the “blue bus – red bus” problem in mode choice.

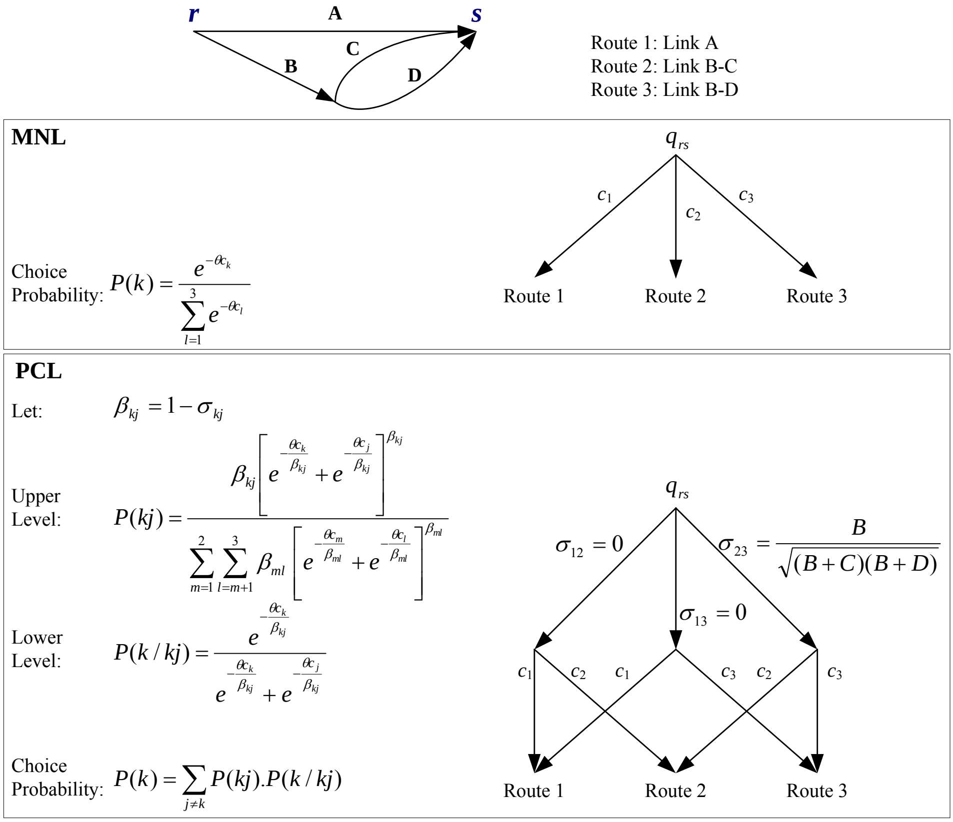


FIGURE 1: Comparison of the MNL and PCL models

It is well known that the simple, one-level tree structure of the MNL model is incapable of distinguishing the different degrees of overlap between route 2 and route 3 when the cost of all three routes are the same. As a result, MNL gives equal probabilities for all three routes. In contrast, the PCL choice probability is evaluated by a hierarchical tree structure, consisting of two levels to reflect the marginal and conditional probabilities. In the upper level, the similarity between each pair of routes is calculated. The similarity index (σ_{kj}) indicates the overlapping portion between a pair of routes (kj). Here we adopt the similarity index suggested by Prashker and Bekhor (2004) by setting the gamma parameter to 1. Other specifications are also possible (see Gliebe et al. (1999) and Prashker and Bekhor (2004)). However, it is important to note that this index must be flow independent (i.e., based on distance or free-flow travel time) in order to retain its analytical tractability. Based on the similarity index and the systematic utility (i.e., route cost $V_k = -\theta c_k$), the marginal probability of choosing route pair kj among three route pairs can also be computed in the upper level. In the lower level, the conditional probability of choosing route k for a given route pair kj is simply a binary logit model, which again can be computed based on the similarity index and the systematic utility. Then, the choice probability of choosing route k can be obtained by multiplying the marginal probability of choosing route pair *kj* with the conditional probability of choosing route *k* for a given route pair *kj* and summing all *j* that is not equal to *k*. As an example, the probability of choosing route 1 is computed as follow: P(1) = P(12)·P(1|12) + P(13)·P(1|13).

As discussed by Bekhor and Prashker (1999), the similarity index computed for each route pair is a highly desirable feature for modeling the route choice problem, because it can represent the distinct overlapping portion of each pair of routes (i.e., each route pair has its own nest in the upper level of the two-level tree structure). In addition, the PCL model reduces to the MNL model when σ_{kj} is equal to zero for all route *kj* pairs (Koppelman and Wen, 2000). In this situation, the routes in the path set are completely disjoint (i.e., no link in common for all route pairs).

2.2 Origin-destination specific scaling factor

The MNL model has the independence from irrelevant alternatives (IIA) property, which is a consequence of the independent and identically distributed (IID) assumptions inherited in the Gumbel assumption of the random error term. The independent assumption in the context of route choice means the overlapping problem. To a certain extent, this can be overcome by the similarity index introduced to the hierarchical tree structure of the PCL model. The identically distributed assumption means that the perception variance with respect to trips of different lengths is constant over all routes for all O-D pairs. In the context of traffic assignment, this assumption seems unrealistic since it does not distinguish trip lengths of different O-D pairs. As suggested by Gliebe et al. (1999), the variance of the Gumbel error term that underlies both the MNL and PCL models can be scaled by individual O-D pair as shown in equation (6)

$$Var(\varepsilon_k^{rs}) = \frac{\pi^2}{6\mu_{rs}'^2}, \tag{6}$$

where μ'_{rs} is a scaling parameter for O-D pair rs. It is important to note that all routes between O-D pair rs are assumed to have the same variance, because scaling each individual route with a different scaling factor would lose the analytical tractability of the MNL and PCL models (i.e., no closed-form probability expression). A common specification of the variance of the perceived route travel times is the summation of perceived travel times on all links comprising it (Sheffi, 1985).

$$Var(\varepsilon_k^{rs}) = \sum_a \beta t_a \delta_{ka}^{rs}, \tag{7}$$

where β is the proportionality constant (assume to be 1.0) and t_a is the travel time on link a. Hence, the O-D specific scaling factor can be computed by equation (8).

$$\mu'_{rs} = \frac{\pi}{\sqrt{6\left(\sum_a \beta t_a \delta_{\bar{k}a}^{rs}\right)}}, \tag{8}$$

where $\bar{k}$ is the minimum travel time route between O-D pair rs. This individual O-D scaling factor would allow the perception variance to increase or decrease according to the travel time of the O-D pair. So, longer O-D pairs would have a larger perception variance compared to those of the shorter O-D pairs. Using the O-D specific scaling factor defined in equation (8), the systematic utility in the MNL and PCL models can be scaled appropriately to reflect different O-D trip lengths in a network as shown in equations (9) and (10).

$$P_k^{rs} = \frac{e^{-\mu_{rs} c_k^{rs}}}{\sum_l e^{-\mu_{rs} c_l^{rs}}}, \tag{9}$$

$$P_k^{rs} = \frac{\sum_{j \neq k} \beta_{kj}^{rs} \, e^{\frac{-\mu_{rs} c_k^{rs}}{\beta_{kj}^{rs}}} \left(e^{\frac{-\mu_{rs} c_k^{rs}}{\beta_{kj}^{rs}}} + e^{\frac{-\mu_{rs} c_j^{rs}}{\beta_{kj}^{rs}}} \right)^{\beta_{kj}^{rs} - 1}}{\sum_{m=1}^{|K_{rs}|-1} \sum_{l=m+1}^{|K_{rs}|} \beta_{ml}^{rs} \left(e^{\frac{-\mu_{rs} c_m^{rs}}{\beta_{ml}^{rs}}} + e^{\frac{-\mu_{rs} c_l^{rs}}{\beta_{ml}^{rs}}} \right)^{\beta_{ml}^{rs}}}, \tag{10}$$

where $\mu_{rs} = \theta \mu'_{rs}$. Note the presence of the O-D specific scaling factor μ'_{rs} in the MNL and PCL models cannot be estimated apart from the dispersion parameter θ. Typically, the scaling factor is arbitrarily set to one for convenience to maintain the assumption of homoscedastic disturbances (Ben-Akiva and Lerman, 1985). However, for traffic assignment purposes where there are many O-D pairs in a network, an O-D specific scaling factor (μ'_{rs}) will be useful in distinguishing O-D pairs of different trip lengths. Instead of setting the scaling factor equal to one, we set the dispersion parameter to one.

3. EXPERIMENTAL SETUP

In order to evaluate the capabilities of the PCL model, the same three-route network shown in Figure 1 is used. The demand is set at 200 vph, the link capacity is set at 100 vph for all links, the dispersion parameter is set to 1.0, and the standard Bureau of Public Roads (BPR) function is adopted to model the congestion effect. Three experimental tests were designed to isolate the scaling effect, the overlapping effect, and the combined effects of scaling and overlapping. Each test will examine two network characteristics with and without congestion effect, resulting in four cases per test. There are a total of twelve cases in all three tests. Table 1 provides a summary of the different effects being examined in the three tests.

TABLE 1: Summary of the effects examined in the three tests

Included?	Effect of		
	Congestion	Scaling	Overlapping
No	Free-flow travel time	Without scaling factor	MNL model
Yes	BPR function	With scaling factor	PCL model

3.1 Test A: The scaling effect

In order to purely test on the identically distributed variance assumption (or scaling effect on congested and uncongested networks), the free-flow travel time on link B is set to zero to create a network with three independent paths (i.e., no overlapping problem). In this case, both PCL and MNL models give the same results since the similarity indices for all route pairs are zero. In addition, two sets of free-flow travel times (FFTT) are created such that the travel times between the two route pairs (i.e., c_2-c_1 and c_3-c_1) differ exactly by one minute. An appropriate model should be able to distinguish between the two settings of FFTT (i.e., longer and shorter O-D pairs with the same route cost difference). Summarized results of cases 1 to 4 in test A are provided in Table 2.

TABLE 2: Probability of choosing route 1: the scaling effect

Case	Effect of			FFTT of link A-B-C-D	
	Congestion	Scaling	Overlapping	4-0-5-5	9-0-10-10
1	No	No	No	0.5761	0.5761
2	No	Yes	No	0.4870	0.4340
3	Yes	No	No	0.4721	0.4307
4	Yes	Yes	No	0.4379	0.3970

In case 1, when none of the three effects is included, the probability of choosing route 1 for both sets of FFTT (4-0-5-5 and 9-0-10-10) is the same. This is because the MNL model assigns probabilities based on absolute differences in route travel times and does not reflect the relative differences. This result reveals the inability of the MNL model to account for the perception variance with respect to trips of different lengths. To reflect the trip lengths of the two sets of FFTT, case 2 includes the scaling effect by computing equation (8) μ_{rs} = 0.6413 for 4-0-5-5, and μ_{rs} = 0.4275 for 9-0-10-10. The result shows that by scaling, the MNL model decreases the probability of choosing route 1 from 0.5761 to 0.4870 for 4-0-5-5 and from 0.5761 to 0.4340 for 9-0-10-10. In addition, scaling seems to be able to distinguish between the two sets of FFTT by assigning different probabilities (0.4870 vs. 0.4340) to reflect the differences in overall trip lengths. Case 3 displays the congestion effect represented by a BPR travel time function on the probabilities of choosing a route in the network. Interestingly, a smooth travel time function like BPR is capable of providing a similar outcome as in case 2. However, note that the route travel times are no longer differed by one minute. The route travel time differences are now determined by the BPR function through equilibrating flows and travel times among the three routes as follow: c_2-c_1 = c_3-c_1 = 0.5814 for 4-0-5-5, and c_2-c_1 = c_3-c_1 = 0.4142 for 9-0-10-10. Case 4 provides results for the combined effect of scaling and congestion. It appears that the combined effect can further reduce the probability of choosing route 1 compared to the isolated effect of either scaling or congestion alone.

3.2 Test B: The overlapping effect

Test B is developed to purely examine the ability of the models to handle the path overlapping problem. In order to accomplish this, the FFTT on link B is set to 3 and 4 to create different degrees of overlap between route 2 and route 3. Note that all three routes are purposely created to have the same route travel time under the free-flow condition. An appropriate model should be able to distinguish between the two settings of overlaps. Scaling is neglected in this test. Summarized results of cases 5 to 8 in test B are provided in Table 3.

In case 5, when none of the three effects is included, the probability of choosing route 1 for both sets of FFTT (5-3-2-2 and 5-4-1-1) is the same. This is because the MNL model treats the three routes as if they were independent and assigns equal probabilities to routes with equal travel times. This result reveals the inability of the MNL model to account for overlapping among routes. To account for the overlapping problem, case 6 adopts the PCL model. The PCL model accounts for the overlapping problem through its hierarchical tree structure by considering the similarity between each route pair. As a result, the probability of choosing route 1 is higher compared to those in case 5 (0.3333 vs. 0.4417 and 0.3333 vs. 0.4728). In addition, the PCL model appears to be able to distinguish the degree of overlap between the two sets of FFTT by assigning different probabilities to route 1 (0.4417 for 60% overlap vs. 0.4728 for 80% overlap). Case 7 displays the congestion effect with route overlap on the route choice probabilities. Similar to case 3, the differences in route travel times are no longer equal zero. These differences under the congestion effect are c_2-c_1 = c_3-c_1 = 0.4022 for 5-3-2-2, and c_2-c_1 = c_3-c_1 = 0.4674 for 5-4-1-1. Case 8 provides results for the combined effect of overlap and congestion. It appears that the combined effect can further increase the probability of choosing route 1 compared to those of the isolated effect of either overlap (case 6) or congestion (case 7) alone.

TABLE 3: Probability of choosing route 1: the overlapping effect

Case	Effect of			FFTT of link A-B-C-D	
	Congestion	Scaling	Overlapping	5-3-2-2	5-4-1-1
5	No	No	No	0.3333	0.3333
6	No	No	Yes	0.4417	0.4728
7	Yes	No	No	0.4278	0.4438
8	Yes	No	Yes	0.4620	0.4832

3.3 Test C: Combined scaling and overlapping effects

In test C, both scaling and overlapping effects are examined simultaneously. The first set of FFTT is the same as in test B, while the second set of FFTT is created to examine the effect of changing the scale of the network characteristics by increasing the FFTT of the first set by a factor of two. An appropriate model should be able to distinguish between the two settings of FFTT with the same fraction of overlapping portion. Summarized results of cases 9 to 12 in test C are provided in Table 4.

TABLE 4: Probability of choosing route 1: the combined scaling and overlapping effects

Case	Effect of			FFTT of link A-B-C-D	
	Congestion	Scaling	Overlapping	5-4-1-1	10-8-2-2
9	No	Yes	No	0.3333	0.3333
10	No	Yes	Yes	0.4728	0.4728
11	Yes	Yes	No	0.4232	0.4356
12	Yes	Yes	Yes	0.4812	0.4824

In case 9, the scaled MNL model assigns equal probabilities to both sets of FFTT since both have equal route travel times. Compared to case 5, scaling has no effect when route travel times are equal. This result reveals that simply scaling the MNL model cannot resolve the overlapping problem. In case 10, the scaled PCL model accounts for the overlap problem between route 2 and route 3 by assigning a higher probability to route 1 compared to case 9, but it assigns the same probability to both sets of FFTT since all three routes have equal travel times in both settings (i.e., the scaling value has no effect on the loading). In case 11, congestion effect is included in the scaled MNL model. Similar to cases 3 and 7, congestion effect changes the probabilities by modifying the route travel time differences. Finally, case 12 provides the results for the combined effects of congestion, scaling, and overlapping. It appears that the combined effect further increases the probability of choosing route 1 compared to the combined effect of either scaling and overlapping or congestion and scaling alone.

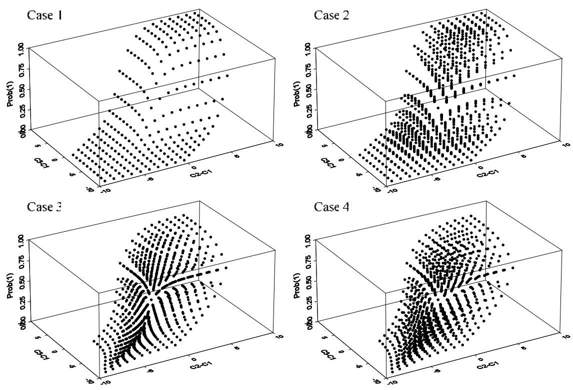
3.4 Further analysis of the twelve cases

This section further analyzes the twelve cases in the three experiments conducted above by investigating many combinations of FFTT settings instead of only two settings as in Tables 2 to 4. Figure 2 depicts the choice probability of route 1, Prob(1), as a function of the differences among the routes' travel times, c_2-c_1 and c_3-c_1. For each plot in Figure 2(a), there are 1,000 combinations of FFTT settings by varying the link free-flow travel time settings of links A, C, and D from 1 to 10 with an increment of 1, and by setting link B to zero to maintain independency among the three routes. The results in Figure 2(a) extends the analysis to other route cost differences in test A. Note that the two settings in Table 2 are only two specific points in each plot of Figure 2(a) where FFTT2 – FFTT1 = FFTT3 – FFTT1 = 1. Figure 2(b) extends the analysis of test B. Each plot consists of 600 points by varying the link free-flow travel time settings. FFTT of link A ($FFTT_A$) is fixed at 5, $FFTT_B$ is varied from 2.5 to 5.0 with an increment of 0.5, and $FFTT_C$ and $FFTT_D$ are varied from 0.5 to 5.0 also with an increment of 0.5. Similarly, Figure 2(c) extends the analysis of test C using the 5-4-1-1 setting in Table 4. Because the same network is used in both tests B and C, the FFTT settings are the same as in Figure 2(b).

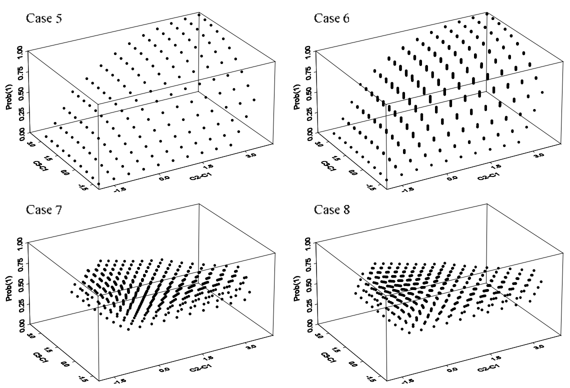
From Figure 2, the general trend in all twelve cases shows an increase in Prob(1) as c_2-c_1 and c_3-c_1 increase (i.e., c_1 becomes smaller, relative to c_2 and c_3). When none of the three effects is included, there are many combinations of route travel time difference that can produce the same probability. Hence, each point in case 1 and case 5 contains many combinations overlaying each other. This observation reveals that the MNL model is indeed incapable of handling the relative difference in trip lengths and the overlapping problem inherited from the independently and identically distributed assumptions. Scaling and/or handling the overlapping problem via the PCL model appear to be able to partially resolve the IID assumptions by modifying the probabilities through a vertical shift (see cases 2, 6, 9, and 10). Congestion effect, on the other hand, changes the route travel time differences by modifying the probabilities through a horizontal shift (see cases 3 and 7); i.e., each setting gives a distinctive route travel time difference; hence, Prob(1) is also distinctive. Combining scaling and/or overlapping effects with congestion effect can further modify the probabilities through both vertical and horizontal shifts (see cases 4, 8, 11, and 12). These graphical illustrations provide intuitive explanation and comparison of the different effects on the calculation of the route choice probabilities.

Based on the above 3-D results, we selected case 1 and case 4 for further analysis. Figure 3 shows the number of combinations in each plot for the two cases. The total number of combinations by summing up the numbers in all cells in Figure 3 is 1000 combinations. Figures 4 and 5 depict the projections of the 3D plots for cases 1 and 4, respectively. Consider the projection on the c_2-c_1 plane of case 1 as depicted in Figure 4, there are a total of 90 combinations that have FFTT2 – FFTT1 = 1 (summing up the numbers under the column FFTT2 – FFTT1 = 1 in Figure 3). Similarly, there are also a total of 90 combinations that have FFTT3 – FFTT1 = 1 (summing up the numbers across the row FFTT3 – FFTT1 = 1 in Figure 3) for the projection on the c_3-c_1 plane of case 1. These projections give different views of the combinations that construct the 3D surface in Figure 2(a). When the combinations are on top of each other, the projections show that these combinations align themselves in a single line (the shaded dots in Figure 4) on

(a) Test A: Cases 1 to 4



(b) Test B: Cases 5 to 8



(c) Test C: Cases 9 to 12

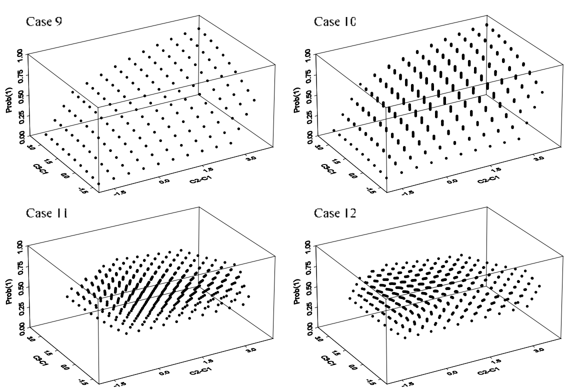


FIGURE 2: Probability of choosing route 1: (a) Test A, (b) Test B, and (c) Test C

FFTT 2 - FFTT 1

FFTT 3 - FFTT 1	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	FFTT 3 - FFTT 1
9										1	1	1	1	1	1	1	1	1	1	9
8									1	2	2	2	2	2	2	2	2	2	1	8
7								1	2	3	3	3	3	3	3	3	3	2	1	7
6							1	2	3	4	4	4	4	4	4	4	3	2	1	6
5						1	2	3	4	5	5	5	5	5	5	4	3	2	1	5
4					1	2	3	4	5	6	6	6	6	6	5	4	3	2	1	4
3				1	2	3	4	5	6	7	7	7	7	6	5	4	3	2	1	3
2			1	2	3	4	5	6	7	8	8	8	7	6	5	4	3	2	1	2
1		1	2	3	4	5	6	7	8	9	9	8	7	6	5	4	3	2	1	1
0	1	2	3	4	5	6	7	8	9	10	9	8	7	6	5	4	3	2	1	0
-1	1	2	3	4	5	6	7	8	9	9	8	7	6	5	4	3	2	1		-1
-2	1	2	3	4	5	6	7	8	8	8	7	6	5	4	3	2	1			-2
-3	1	2	3	4	5	6	7	7	7	7	6	5	4	3	2	1				-3
-4	1	2	3	4	5	6	6	6	6	6	5	4	3	2	1					-4
-5	1	2	3	4	5	5	5	5	5	5	4	3	2	1						-5
-6	1	2	3	4	4	4	4	4	4	4	3	2	1							-6
-7	1	2	3	3	3	3	3	3	3	3	2	1								-7
-8	1	2	2	2	2	2	2	2	2	2	1									-8
-9	1	1	1	1	1	1	1	1	1	1										-9
	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	

FFTT 2 - FFTT 1

FIGURE 3: Number of combinations in Case 1 and Case 4

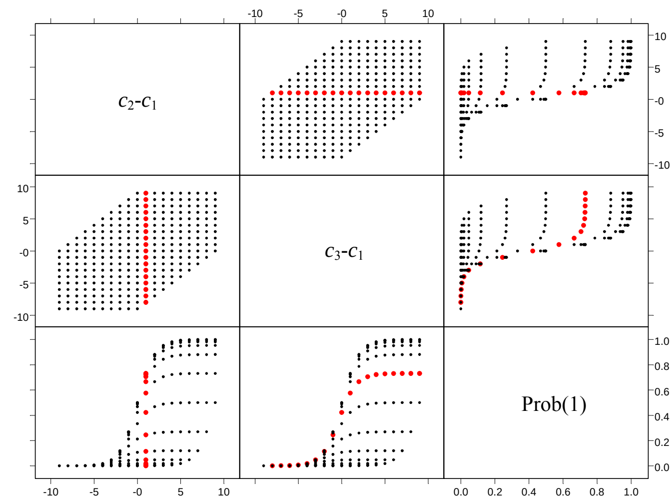


FIGURE 4: Projections of Case 1

the c_2-c_1 and c_3-c_1 planes. Under this situation, the MNL model is incapable of distinguishing the different combinations and assigns the same probability for all these combinations that are overlaying each other. When both congestion and scaling effects are included as in case 4, these combinations are dispersed through a horizontal movement (congestion effect) and a vertical movement (scaling effect) as shown in the 3D surface in Figure 2(a). Again, Figure 5 provides different views of the dispersion effect by projecting the 3D surface to different planes. It shows that the 90 combinations are no longer aligned in a straight line on the c_2-c_1 and c_3-c_1 planes. This also changes the choice probability on route 1. In effect, including congestion and scaling effects can partially resolve the drawbacks of the MNL model (i.e., trips of different lengths).

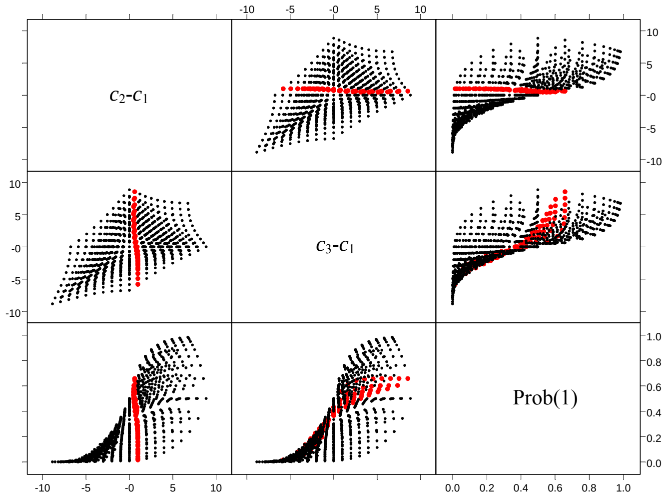


FIGURE 5: Projections of Case 4

Now, consider the cell where both FFTT2 – FFTT1 and FFTT3 – FFTT1 equal to 1 (shaded in Figure 3). There are nine combinations (i.e., 1-0-2-2, 2-0-3-3, 3-0-4-4, “4-0-5-5”, 5-0-6-6, 6-0-7-7, 7-0-8-8, 8-0-9-9, and “9-0-10-10”) in this cell overlaying each other. For case 1 (i.e., none of the three effects is included), the probability of choosing route 1 is the same (0.5761) for all nine combinations (shown in bold in Figure 6) since they all have the same absolute difference in route travel times. These results extend those presented in Table 2 for the two specific combinations “4-0-5-5” and “9-0-10-10”, and confirm the inability of the MNL model to account for the perception variance with respect to trips of different lengths. When both congestion and scaling effects are included, these nine combinations start to disperse symmetrically and yield different probabilities of choosing route 1. This is attributed to the congestion effect (i.e., absolute difference in route travel times is different for each combination; they are no longer equal to 1 as in case 1) and to the scaling effect (i.e., longer trip lengths have a lower probability). For clarity, we also present Figure 7 to show the projections of Figure 6 on the c_2-c_1 and c_3-c_1 planes. Since the network is symmetrical, the c_2-c_1 and c_3-c_1 planes are plotted on the same x-axis, and the probability of choosing route 1 is shown on the y-axis. In this figure, we can observe clearly how the nine combinations in case 1 (all with the same probability of choosing route 1) disperse to different probabilities according to their trip lengths and route cost differences. Using the 9-0-10-10 setting as an example, the probability of choosing route 1 changes from 0.5761 in case 1 to 0.3970 in case 4, and the route cost difference changes from 1.000 in case 1 to 0.6621 in case 4. This indicates that, when combined with congestion effects, the variance scaling factor could provide results that are significantly different from the simple MNL model.

4. CONCLUSIONS

This paper has shown how the PCL model resolves the drawbacks inherited in the classical MNL model using a set of carefully designed experiments and graphical illustrations to examine the effects of scaling, overlapping, and congestion. The

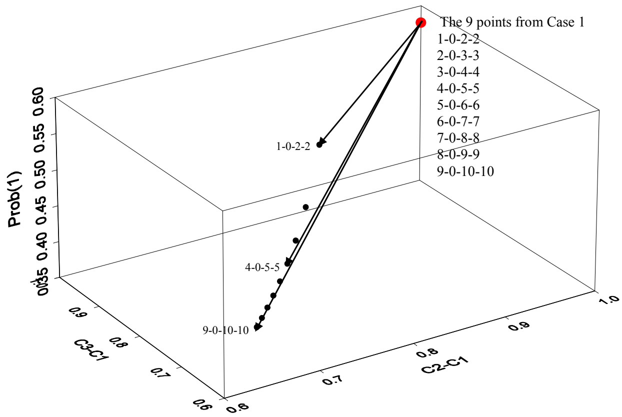


FIGURE 6: Movements in Case 4 relative to Case 1

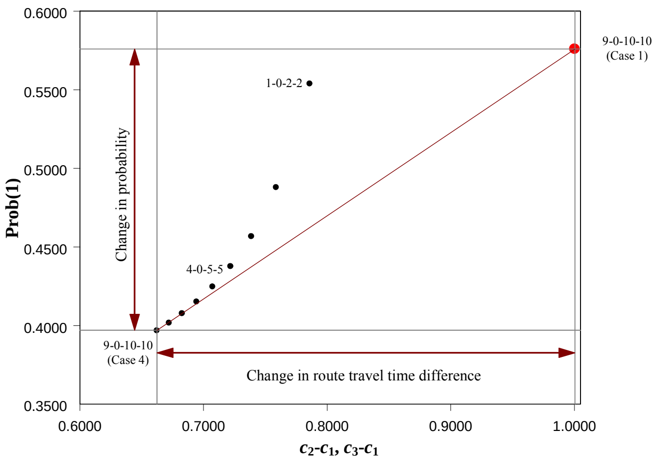


FIGURE 7: Projection of movements in Case 4 relative to Case 1

independent assumption is handled through the similarity index introduced in the hierarchical tree structure of the PCL model; the identically distributed assumption is partially relaxed through the introduction of O-D specific scaling factor; and congestion can further modify the probability through equilibrating flows and travel times via a link travel time function. However, it has been pointed out in the literature that when demand is high, the congestion effect dominates over the other effects. Overall, it appears that the scaled PCL model combined with congestion effect has good features that can be used to model the route choice problem. Further experiments on realistic networks are necessary to substantiate the results presented in this paper. Future work should also include collecting route choice data for model estimation.

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