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OPERATIONAL EFFECTS OF ACCELERATION LANE ON MAIN TRAFFIC FLOW AT DISCONTINUITIES

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Recent studies have indicated that on-ramp flow is important in the formation of the stop-and-go traffic flow near the ramp. Several models have been developed to explain the complex phenomena associated with ramps which result in the hysteretic phase transitions. A good understanding of the mechanisms of congestion near on-ramps is very useful for the development of suitable control and highway design measures. To this end, in this paper traffic flow operations on a freeway with an on-ramp are investigated based on a newly developed second order macroscopic model. The proposed model allows to take into account the influences of the acceleration lane length on the flow dynamics of the main carriageway. The main aim of this paper is, therefore, to investigate analytically and numerically the effects of acceleration lane length on traffic flow operations. It is found that different acceleration lane lengths result in various congested traffic states.

KEYWORDS: macroscopic model, on/off ramp, model validation, linear stability analysis

1. INTRODUCTION

Since Lighthill and Whitham (1955), and Richards (1956) first applied a simple (continuum) macroscopic model to describe the evolution of traffic flow on freeways, significant efforts have been undertaken to further develop and employ macroscopic traffic flow models. These efforts have, however, mainly concentrated on describing uninterrupted traffic flow. In contrast, relatively little progress has been made in the investigation of traffic flow with discontinuities such as at on-and off-ramps, weaving sections, and so on. In the macroscopic models, traffic operations at on-and off-ramps are often inadequately treated, which are generally related to oversimplification. For example, most models neglect the length of the acceleration lane and, instead, treat a ramp as a singular point ('point-like model'). In those models, the inflow from the on-ramp is often included in one section. In order to take into account the influences of the merging flow on the main flow, the so-called traffic friction is added to the right hand side of some continuum (macroscopic) models. Examples of these point-like models can be seen in Cremer and Ludwig (1986), Papageorgiou et al. (1989), Michalopoulos et al. (1993), Liu et al. (1996). Other models are obtained from distinct approaches such as the ramp model of Kerner et al. (1995). In this model, the acceleration lane length is neglected and the impact of ramp flow is only considered in the conservation equation. The ramp model of Helbing et al. (1999a) takes into account the effects of the acceleration lane length on the dynamics of the main traffic flow in a deterministic way. Simulation results of the model of Helbing et al. (1999a) indicated that the longer ramps lead to the higher roadway capacities. The influences of the acceleration lane length on the traffic flow dynamics have been captured explicitly in microscopic models, for example by Hidas (2005), but not yet in the (continuum) macroscopic models.

Recent studies have indicated that impact of on-ramp flow is important in the formation of the stop-and-go traffic flow near the ramp. Several models have been developed to explain the complex phenomena associated with ramps such as moving clusters, stop-and-go waves, oscillating and homogeneous congested states, and so on.

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Ngoduy (2006a) and Ngoduy et al. (2006) have introduced a generalized continuum model that is able to describe the lane-changing behaviour at discontinuities including on-and off-ramps, weaving sections based on a *gap-acceptance* approach. Ngoduy et al. (2006) have shown that the introduced model is also able to replicate different congested traffic states that often occur on freeways. This property is consistent with the findings of Helbing and Treiber (1998), Treiber et al. (2000). Based on this newly developed model, in this paper, an investigation of the factors that influence traffic flow operations at discontinuities is performed analytically and numerically. We show that, among the other things, the acceleration lane length, which has been neglected when analyzing the stability of other (continuum) macroscopic traffic models, is an important factor that influences the stability of the main carriageway traffic flow. Given the same perturbation of on-ramp traffic flow, the longer the acceleration lane is the more stable the main traffic is.

This paper is organized as follows. Section 2 overviews the new approach to develop a high order macroscopic model for traffic flow operations at discontinuities on freeways. Section 3 presents an analytical investigation on how the acceleration lane length influences the traffic dynamics on the main carriageway. In Section 4, we carry out a numerical study to support qualitatively our analytical results. Finally, we conclude the paper in Section 5.

2. MACROSCOPIC TRAFFIC MODEL ON FREEWAYS WITH DISCONTINUITIES

This section reviews a newly developed macroscopic model describing the traffic dynamics on freeways with discontinuities. The model has been implemented based on the *gap-acceptance* approach using a gas-kinetic traffic flow theory. For more details of the development of the generalized continuum model for multilane and multiclass traffic flow with discontinuities, we refer to Ngoduy (2006a) and Ngoduy et al. (2006). To derive a (continuum) macroscopic model for interrupted traffic flow on freeways, the gas-kinetic modelling is chosen as an intermediate step. The development of gas-kinetic traffic models started in the 1960s with a simple model of Prigogine and Andrews (1960) and Prigogine (1961). In the gas-kinetic models, vehicles and drivers' behaviour are described by means of probability distribution functions. However, the behavioral rules are still described at an individual level. The dynamics of these distributions are generally governed by various processes, such as, acceleration, interactions between vehicles, and lane-changing, describing the individual drivers' behaviour. The gas-kinetic traffic flow models are based on descriptions of the dynamics of the phase-space (or time-location) density, that is, the dynamics of the speed distribution functions of vehicles in the traffic flow. Given the knowledge of the phase profile of density, one can determine the continuum (macroscopic) traffic variables such as density, mean speed, or flow rate, by means of the *method of moments*. Let $\rho(x,v,t)$ denote the phase-space density (*PSD*) function, which is interpreted as follows: at instant time t the expected number of vehicles present at a small cell $[x,x+\mathrm{d}x]$ driving with a speed in the region $[v,v+\mathrm{d}v]$ is equal to $\rho(x,v,t)\mathrm{d}x\mathrm{d}v$. Based on the conservation law, the equation for the dynamics of $\rho(x,v,t)$ can be found. Further development of the gas-kinetic models for uninterrupted traffic flow can be found in Paveri-Fontana (1975), Helbing (1996), Helbing et al. (1999b), Hoogendoorn and Bovy (1999), and Shvetsov and Helbing (1999). In this section, we introduce a newly developed gas-kinetic model for freeway traffic flow with discontinuities (e.g. with on- and off-ramps). It is worth mentioning that there is another approach to describe merging and diverging behavior based on the LWR model (see Ni et al. (2006) and references there-in). However, this paper mainly focuses on the performance of the newly developed second order model of Ngoduy et al. (2006). To describe traffic operations in the main lane and the acceleration lane (that is, either an on-ramp lane or off-ramp lane) we need to determine the mandatory lane-changing (MLC) rate between these lanes. In general, the MLC rate is inversely proportional to the remaining distance to the end of the ramp. That is, the closer to the end of the ramp, the higher the lane-changing rate from the acceleration lane to the main lane or from the main lane to the off-ramp. This guarantees that all vehicles have changed lanes at the end of the acceleration lane.

Let $\tilde{\nu}^{\pm}(x,v,t)$ denote the incoming (plus sign) and exiting (minus sign) flow rate to and from the main carriageway, respectively, at location x and time instant t. Let $\rho_0(x,v,t)v$ define the expected volume of vehicle driving with speed v and having to merge at location x and time instant t. The expected MLC flow rates are determined as follows:

In merging case:

$$\tilde{\nu}^{+}(x,v,t) = \delta(x)\frac{\rho_0(x,v,t)v}{L}\pi_{0,1}\,. \tag{1}$$

In diverging case:

$$\tilde{\nu}^{-}(x,v,t) = -\delta(x)\frac{\alpha_{1,0}\rho_(x,v,t)v}{L}\pi_{1,0}\,. \tag{2}$$

Note that in the rest of this paper, the lane index 0 indicates the acceleration lane whereas the lane index 1 denotes the main carriageway lane. Accordingly, $\pi_{0,1}$ and $\pi_{1,0}$ denote the expected probability to merge and diverge, respectively, from the acceleration lane to the main carriageway lane and from the main carriageway lane to the off-ramp. $\alpha_{1,0}$ is the fraction of traffic flow that intends to exit the main carriageway to the off-ramp. $\delta(x)$ = 0 if x is outside of the ramp area, $\delta(x)$ = 1 otherwise. L denotes the acceleration lane length.

The gas-kinetic model for (aggregate lane and aggregate vehicle class) traffic stream with discontinuities reads (Ngoduy, 2006a):

$$\frac{\partial\rho}{\partial t} + \underbrace{v\frac{\partial\rho}{\partial x}}_{\text{convection}} + \underbrace{\frac{\partial}{\partial v}\left(\rho\frac{V_{\max}-v}{\tau}\right)}_{\text{acceleration}} = \underbrace{\left(\frac{\partial\rho}{\partial t}\right)_{\text{int}}}_{\text{interaction}} + \underbrace{\tilde{\nu}^{+}(x,v,t)+\tilde{\nu}^{-}(x,v,t)}_{\text{mandatory lane change}}\,, \tag{3}$$

where $V_{\max}$ denotes the free speed and τ is the relaxation time. In equation (3) the *convection* term represents the changes of the phase space density (*PSD*) due to the longitudinal movement of vehicles while the *acceleration* term accounts for the changes of the *PSD* due to the acceleration process of drivers to their desired speed. The *interaction* term describes the changes of the *PSD* due to the interaction between faster vehicles and slower vehicles. In general, faster vehicles catch up with slower vehicles and, consequently, have to decelerate to the slower speed. The *mandatory lane changing* term models the changes of the *PSD* due to the inflow and outflow at on- and off-ramps. From equation (3), we can obtain the corresponding macroscopic model for traffic flow with discontinuities using the so-called *method of moments*. More details of the derivation using the *method of moments* are in Ngoduy(2006a) and Ngoduy et al. (2006). The resulting macroscopic model reads:

Conservation law:

$$\frac{\partial r}{\partial t}+\underbrace{\frac{\partial q}{\partial x}}_{\text{convection}}=\underbrace{\delta(x)\frac{q_0}{L}\pi_{0,1}-\delta(x)\frac{\alpha_{1,0}q}{L}\pi_{1,0}}_{\text{mandatory lane change}}. \quad (4)$$

Momentum dynamics:

$$\frac{\partial q}{\partial t}+\underbrace{\frac{\partial r\left(V^2+\Theta\right)}{\partial x}}_{\text{convection}}=\underbrace{\frac{r\left(V^{\mathrm{e}}-V\right)}{\tau}}_{\text{acceleration}}+\underbrace{\delta(x)\frac{q_0V_0}{L}\pi_{0,1}-\delta(x)\frac{\alpha_{1,0}qV}{L}\pi_{1,0}}_{\text{mandatory lane change}}. \quad (5)$$

In equations (4) and (5), r, V, q and Θ denotes, respectively, the density, mean speed, flow and mean speed variance. q_0 and V_0 are, respectively, the expected flow and mean speed from the on-ramp. V^{e} denotes the equilibrium mean speed and Θ is the speed variance of the main carriageway traffic. To close equations (4) and (5), Θ is empirically determined as a function of the speed and density. In this paper, we adopt the following empirical relationships of the equilibrium speed and speed variance as proposed by Treiber et al. (1999), Shvetsov and Helbing (1999) and Helbing et al. (1999a).

$$\Theta=\Theta^{e}\left(r,V\right) \text{ and } V^{\mathrm{e}}=V^{\mathrm{e}}\left(r,V,\Theta\right). \quad (6)$$

The specific formulation of Θ and V^{e} is given in Section 4.

The expected merging and diverging probabilities $\pi_{0,1}$ and $\pi_{1,0}$, respectively, have been determined using the *gap-acceptance* model and the *Renewal theory* in Ngoguy (2006a) and Ngoduy et al. (2006). These probabilities turn out to be dependent on many factors such as traffic variables (e.g. density, speed, speed variance), safe time headway and acceleration lane length. The performance of our model with respect to real-life data has been investigated in Ngoduy (2006b). In the ensuing sections, we will investigate analytically and numerically the effects of the acceleration lane length on traffic flow operations at freeway discontinuities.

3. ANALYTICAL INVESTIGATION OF THE ACCELERATION LANE EFFECTS

This section investigates analytically the property of the introduced model to describe traffic flow operations at freeway discontinuities. In order to do so, we derive stability conditions based on the linear method for the introduced (macroscopic) model. The linear method refers to linear Taylor approximations, which are used throughout the analysis. The consequence of these approximations is that the conditions that are stable according to this analysis might actually still show non-linear instability. However, in general the linear analysis gives sound insights in the general behaviour of the model in the presence of on- and off-ramps. To facilitate the stability analysis of the proposed model but without loss of generality, we only focus on the on-ramp model while it is not necessary to also show the results for the off-ramp model, because in the off-ramp model, we only change the sign of the ramp-flow demand, that is, the flow rate desired to exit. Let us first start with the linear stability analysis of the model with the main focus on disturbances due to traffic from the on-ramp and the effects of the acceleration lane length on distributing and spreading out these disturbances. Let us rewrite equations (4) and (5) in the following formula:

$$\frac{\partial r}{\partial t}+V\frac{\partial r}{\partial x}+r\frac{\partial V}{\partial x}=\frac{q_0}{L}\pi, \quad (7)$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{1}{r}\frac{\partial r\Theta}{\partial x}=\frac{V^{\mathrm{e}}-V}{\tau}+\frac{q_0\left(V_0-V\right)}{rL}\pi\,. \tag{8}$$

Note that $\delta(x)$ and the index (0,1) of the merging probability have been dropped out for the sake of simplicity. Let us suppose that the *stationary* and *spatially* homogeneous solution of the system of equations (7) and (8) are the constant density r_{ss} and corresponding speed $V_{ss} = V^{e}(r_{ss})$. It has been shown in Ngoduy et al. (2006) that at this solution the merging probability $\pi \to 0$. Now let us consider large perturbations around the solution pair (r_{ss} and V_{ss}), denoted by $\delta r(x,t)$ and $\delta V(x,t)$, which lead to:

$$r(x,t)=r_{ss}+\delta r(x,t)\ \text{ and }\ V(x,t)=V_{ss}+\delta V(x,t)\,. \tag{9}$$

The region of instability can be found by the linear stability analysis as described in the rest of this section. Let these perturbations be determined as corresponding cosine functions with frequency λ and wave number ξ as shown below (Helbing, 1997):

$$\delta r(x,t)=\delta r_0 e^{\lambda t+k\xi x}\ \text{ and }\ \delta V(x,t)=\delta V_0 e^{\lambda t+k\xi x}\,, \tag{10}$$

where k denotes the imaginary unit (to differentiate with lane index i we use k instead of the conventional notation), δr_0 and δV_0 are constants.

By substituting equations (9) and (10) into equations (7) and (8), given the acceleration lane length L and ramp inflow q_0, we obtain:

$$\frac{\partial\delta r}{\partial t}+V_{ss}\frac{\partial\delta r}{\partial x}+r_{ss}\frac{\partial\delta V}{\partial x}-\frac{q_0}{L}\left(\frac{\partial\pi}{\partial r}\delta r+\frac{\partial\pi}{\partial V}\delta V\right)=0\,, \tag{11a}$$

$$\frac{\partial\delta V}{\partial t}+V_{ss}\frac{\partial\delta V}{\partial x}+\frac{1}{r_{ss}}\left(\frac{\partial r\Theta}{\partial r}\frac{\partial\delta r}{\partial x}+\frac{\partial r\Theta}{\partial V}\frac{\partial\delta V}{\partial x}\right)-\left(\frac{\partial\phi}{\partial r}\delta r+\frac{\partial\phi}{\partial V}\delta V\right)=0\,, \tag{11b}$$

where ϕ denotes the right hand side of equation (8). If we substitute expression (10) into system (11) we obtain:

$$\underbrace{\begin{pmatrix}\lambda-a_{11} & a_{12}\\ a_{21} & \lambda-a_{22}\end{pmatrix}}_{J}\begin{pmatrix}\delta r\\ \delta V\end{pmatrix}=0\,, \tag{12}$$

where

$$\begin{aligned}a_{11}&=\frac{q_0}{L}\frac{\partial\pi}{\partial r}-k\xi V_{ss}, & a_{12}&=-\frac{q_0}{L}\frac{\partial\pi}{\partial r}+k\xi r_{ss},\\ a_{21}&=-\frac{\partial\phi}{\partial r}+\frac{1}{r_{ss}}\frac{\partial r\Theta}{\partial r}k\xi, & a_{22}&=\frac{\partial\phi}{\partial V}-\frac{1}{r_{ss}}\frac{\partial r\Theta}{\partial V}k\xi-k\xi V_{ss}.\end{aligned} \tag{13}$$

The system is stable if: (1) the deviation pair (δr and δV) are a solution for equations (7) and (8), and (2) if the amplitude of the perturbation (10) is decreasing with time. The latter is true if the real part of frequency λ is strictly negative. Condition 1 is satisfied if the determinant of the *Jacobian* matrix J is zero, that is:

$$\det\left(J\right)=0\Leftrightarrow\lambda^2-\left(a_{11}+a_{22}\right)\lambda+\left(a_{11}a_{22}-a_{12}a_{21}\right)=0\,, \tag{14}$$

which results in:

$$\lambda_1=0.5\left(a_{11}+a_{22}+\sqrt{\left(a_{11}-a_{22}\right)^2+4a_{12}a_{21}}\right), \tag{15a}$$

$$\lambda_2=0.5\left(a_{11}+a_{22}-\sqrt{\left(a_{11}-a_{22}\right)^2+4a_{12}a_{21}}\right). \tag{15b}$$

The following approximation for the real part R and complex part K of expression (15) is used:

$$\sqrt{R+kK} = \sqrt{0.5\sqrt{R^2+K^2}+R} + k\sqrt{0.5\sqrt{R^2+K^2}-R}\ . \tag{16}$$

Hence,

$$\mathrm{Real}(\lambda_1) = 0.5\left(\frac{q_0}{L}\frac{\partial\pi}{\partial r}+\frac{\partial\phi}{\partial V}+\sqrt{0.5\left(\sqrt{R^2+K^2}+R\right)}\right), \tag{17a}$$

$$\mathrm{Real}(\lambda_2) = 0.5\left(\frac{q_0}{L}\frac{\partial\pi}{\partial r}+\frac{\partial\phi}{\partial V}-\sqrt{0.5\left(\sqrt{R^2+K^2}+R\right)}\right), \tag{17b}$$

where

$$R = \left(\frac{q_0}{L}\frac{\partial\pi}{\partial r}-\frac{\partial\phi}{\partial V}\right)^2 - \left(\frac{\xi}{r_{ss}}\frac{\partial r\Theta}{\partial r}\right)^2 + 4\left(\frac{q_0}{L}\frac{\partial\pi}{\partial V}\frac{\partial\phi}{\partial r}-\xi^2\frac{\partial r\Theta}{\partial r}\right), \tag{18a}$$

$$K = 2\xi\left[\left(\frac{q_0}{L}\frac{\partial\pi}{\partial r}-\frac{\partial\phi}{\partial V}\right)\frac{1}{r_{ss}}\frac{\partial r\Theta}{\partial V}-2\frac{1}{r_{ss}}\frac{q_0}{L}\frac{\partial\pi}{\partial V}\frac{\partial r\Theta}{\partial r}-2r_{ss}\frac{\partial\phi}{\partial r}\right]. \tag{18b}$$

Obviously, condition 2 is strictly fulfilled if $\mathrm{Real}(\lambda_1,\lambda_2)<0$, which corresponds to:

$$\begin{cases} \dfrac{q_0}{L}\dfrac{\partial\pi}{\partial r}+\dfrac{\partial\phi}{\partial V} < -\sqrt{0.5\left(\sqrt{R^2+K^2}+R\right)}, \\ \dfrac{q_0}{L}\dfrac{\partial\pi}{\partial r}+\dfrac{\partial\phi}{\partial V} < +\sqrt{0.5\left(\sqrt{R^2+K^2}+R\right)}. \end{cases} \tag{19}$$

At the *stationary* and *spatially* homogeneous solution, since $V_{ss} = V^e(r_{ss})$, the speed variance Θ is a density-dependent function $\Theta_{ss} = \Theta^e(r_{ss})$; after a lengthy but rather straightforward algebraic calculation, inequality (19) becomes:

$$\left(\frac{q_0}{L}\frac{\partial\pi}{\partial r}+\frac{\partial\phi}{\partial V}\right)^2\frac{\partial r\Theta}{\partial r} > \left(r_{ss}\frac{\partial\phi}{\partial r}+\frac{1}{r_{ss}}\frac{q_0}{L}\frac{\partial\pi}{\partial V}\frac{\partial r\Theta}{\partial r}\right)^2, \tag{20}$$

of which solutions are:

$$\frac{q_0}{L} < \frac{\left|\frac{\partial\phi}{\partial V}\right|}{\left|\frac{\partial\pi}{\partial r}\right|}, \tag{21}$$

$$r < \frac{\left|\frac{\partial\phi}{\partial V}\right|}{\left|\frac{\partial\phi}{\partial r}\right|}\sqrt{\frac{\partial r\Theta}{\partial r}}\ . \tag{22}$$

Example of the function π, ϕ, and Θ to determine the right hand sides of equations (21) and (22) is presented in Section 4.

If there is no on-ramp in the considered freeway and it is assumed that speed variance Θ is a constant as in the model of Payne (1971), condition (21) does not apply while condition (22) becomes:

$$r < \frac{1}{\sqrt{2\tau\left|\frac{\mathrm{d}V^e}{\mathrm{d}r}\right|}}\ . \tag{23}$$

Condition (23) is consistent with the condition derived by Helbing (1997) for the Payne model.

It is clear that in the presence of on-ramps, there are two conditions for stable traffic flow. On the one hand, the condition of the density regime (22) must be satisfied. On the other hand, the condition for the ramp flow and acceleration lane length (21) needs to be fulfilled as well. From condition (21), we can see that, given that condition (22) is satisfied, the fluctuation of the inflow from the on-ramp may cause instability of the main traffic flow which results in congestion. For a fixed acceleration lane length, below a certain critical value of ramp flow, the free traffic state may survive but when the on-ramp flow is higher, traffic becomes unstable, leading to the stop-and-go state, oscillatory congested state, or even homogeneous congested state if the main carriageway is heavily used already. These phenomena have been reported in Helbing and Treiber (1998) and Helbing et al. (1999a). However, the acceleration lane length is also an important factor, which spreads the perturbations of the inflow from on-ramp. Obviously the longer the acceleration lane is, the more stable the main traffic flow is. This finding is consistent with Helbing et al. (1999a), since when the ramp flow goes to zero or the acceleration lane length goes to infinite, condition (21) is always satisfied. That means that the ramp flow does not cause instability on the main carriageway traffic flow operations.

To support the above analytical findings, we will show some numerical outcomes with major focus on the important effects of the acceleration lane on main traffic flow operations.

4. NUMERICAL STUDY

In this section, the model equations (4) and (5) are approximated using a dedicated numerical scheme for second order macroscopic traffic models. The adopted numerical scheme, proposed by Ngoduy et al. (2004), is based on an approximation of the Riemann solver and has shown improvements over existing numerical schemes with respect to second order macroscopic traffic models. We refer to Ngoduy et al. (2004) for more details. In this numerical study, we assume that the aggregate lane road has a length of 10 km and that an on-ramp is located at x = 5 km. The time horizon is 30 minutes. The road is divided into small segments or cells with Δx = 100 m in length; the time step is chosen as Δt = 2 sec. These discretization values are chosen to ensure the Courant-Friedrich-Lewy numerical stability conditions: $\Delta x \geq \Delta t V_{\max}$, where $V_{\max}$ denotes the free speed (Sod, 1985).

4.1 Numerical setup

Let us first provide the functions of speed variance Θ, equilibrium speed V^e and merging probability π in our simulation.

The speed variance is determined by Treiber et al. (1999), Shvetsov and Helbing (1999) and Helbing et al. (1999b):

$$\Theta = A(r)V^2 , \tag{24}$$

where $A(r)$ denotes the so-called variance pre-factor, which is higher in congested traffic than in free traffic. The empirical function of $A(r)$ is:

$$A(r) = \bar{A} + \Delta A\left[1 + \exp\left(\frac{r_{cr} - r}{\Delta r}\right)\right]^{-1}, \tag{25}$$

where $\bar{A}$ and ΔA are the variance pre-factors for free and congested traffic, respectively. r_{cr} is critical density for the transitions from the free-flow to the congested traffic states, and Δr denotes the width of the transitions.

The empirical equilibrium speed is:

$$V^{\mathrm{e}} = \frac{\hat{V}^2}{2V_{\max}}\left(-1 + \sqrt{1 + \frac{4V_{\max}^2}{\hat{V}^2}}\right), \tag{26}$$

where

$$\hat{V} = \frac{1}{T}\left(\frac{1}{r} - \frac{1}{r_{\max}}\right)\sqrt{\frac{A(r_{\max})}{A(r)}}, \tag{27}$$

$r_{\max}$ is the jam density, and T denotes the safe time headway.

The merging probability of traffic from the on-ramp is determined by Ngoduy (2006a) and Ngoduy et al. (2006) as:

$$\pi = \exp\left[-\gamma r\left(\frac{1}{r_{\max}} + TV\right)\right]\exp\left[-\gamma r\left(\frac{1}{r_{\max}} + \mu TV\right)\right]\left[1 + 0.5\Theta(\gamma rT)^2\right], \tag{28}$$

where γ is the space requirement factor that accounts for the impact of the vehicle length in dense traffic, determined by Hoogendoorn et al. (2002):

$$\gamma = \frac{1}{1 - r\left(\frac{1}{r_{\max}} + TV\right)}. \tag{29}$$

The factor μ ($\mu_{\min} \leq \mu \leq \mu_{\max}$) accounts for the *gap-acceptance* behaviour of the driver when approaching the end of the acceleration lane. That is, when approaching the end of the acceleration lane drivers tend to accept smaller gaps ($\mu \to \mu_{\min}$) and disturb the main traffic flow more significantly.

Model parameters for the simulation are given in Table 1. The corresponding equilibrium mean speed and variance pre-factor with respect to the density are illustrated in Figure 1. In this simulation, open-boundary conditions are used in order to see the influences of the inflow from the on-ramp on main traffic flow operations. Since in this paper we mainly focus on analyzing the effects of the acceleration lane length on the main traffic, we set the inflow from upstream of the main carriageway to 2100 veh/h and the inflow from the on-ramp to 200 veh/h. The initial speed of the main flow and the on-ramp flow is set to 70 km/h. The acceleration lane length is set to vary between 50 m, 200 m and 2000 m. It is worth to notice that when L = 50 m, the discretization steps become Δx = 50 m and Δt = 1 sec and the ramp flow is forced to merge in only one cell.

4.2 Simulation results

When the acceleration lane length is set to a typical value 200 m, a well-known congested pattern- so-called Moving Localized Clusters (MLCs) (Helbing and Treiber, 1998) - occurs due to the perturbation of on-ramp flow that has been added to the already heavy main traffic (Figure 2). Note that model equations (4) and (5) are called second

TABLE 1: Model parameters for numerical simulation

Parameters	Notations	Units	Values
Free speed	V_{max}	km/h	100
Jam density	r_{max}	veh/km	150
Critical density	r_{cr}	veh/km	33
Relaxation time	τ	sec	15
Safe time headway	T	sec	1.2
Speed variance pre-factor	$\bar{A}$		0.008
	ΔA		0.03
	Δr	veh/km	5
Merging factor	μ_{min}		0.1
	μ_{max}		0.9

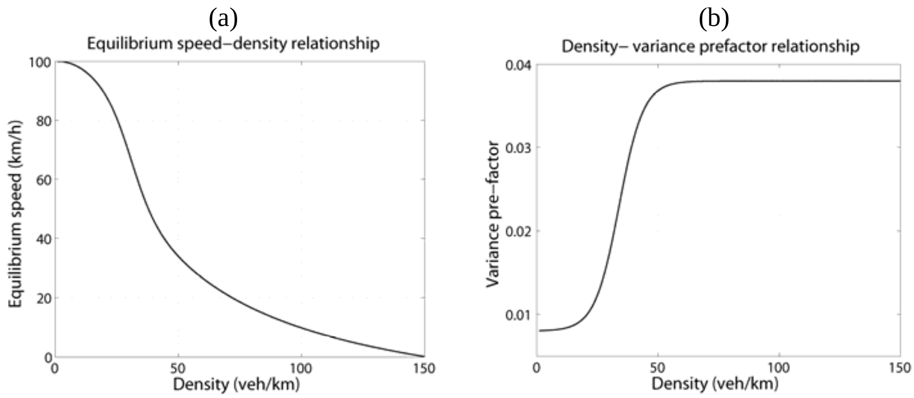

FIGURE 1: Empirical density-dependent relationships

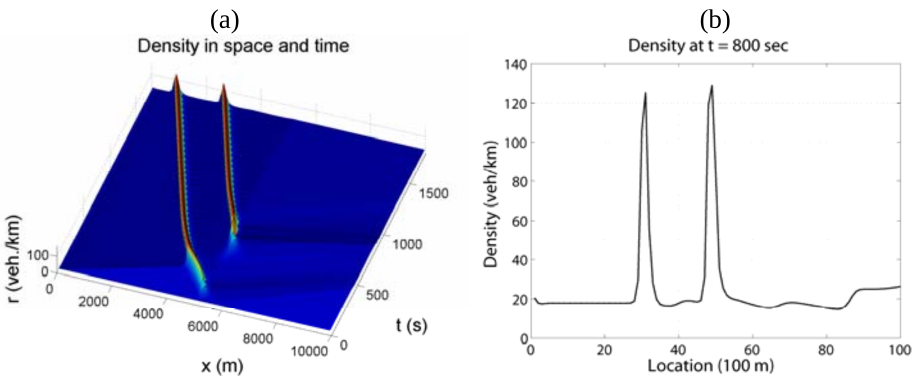

FIGURE 2: Moving Localized Clusters with acceleration lane length = 200 m

order macroscopic traffic model and have two characteristic speeds (denoted as λ_{free} and λ_{cong} in Figure 3). The mathematical analysis of general second order macroscopic models is given in Ngoduy et al. (2004). As it can be seen from Figure 3 the maximum characteristic speed λ_{free} accounts for the disturbances that travel along traffic direction in free-flow conditions while the minimum characteristic speed λ_{cong} represents the

(a) (b)

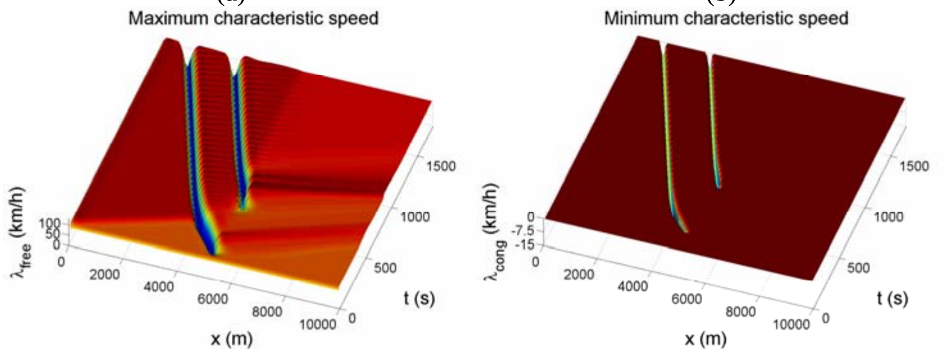

FIGURE 3: Characteristic speeds of Moving Localized Clusters

disturbances that travel against traffic direction in congested traffic states. The cluster travel against traffic direction at speed of around 15 km/h (Figure 3b). When the fist MLC start moving upstream of the on-ramp, it triggers another density cluster that finally grows up to a second MLC. The distance between clusters is around 2 km while the time interval is around 550 sec. The density differences between the congested and free-flow conditions are approximately 100 veh/km (Figure 2b). When the acceleration lane length is too short (L = 50 m), the main traffic is more unstable (more clusters are formed with shorter space and time intervals) due to the same perturbation of the on-ramp flow. The so-called oscillatory congested traffic state (OCT) occurs (Figure 4). In OCT state, the clusters also travel against traffic direction at speed of 15 km/h (Figure 4b). As it can be seen from Figure 5a, the space distance between clusters becomes shorter, which is around 700 m. Similarly, the time interval between clusters is reduced to 300 sec (Figure 5b). The density differences between the congested and free-flow conditions are 120-130 veh/km (Figure 5). If we set the acceleration lane very long (e.g. L = 2000 m), the stale traffic can be obtained as described in Figure 6. In this case, all disturbances travel along traffic direction and no congestion occurs. Last but not least, based on our simulation we found that closer to the end of the acceleration lane, the gradient of the merging probability with respect to the density increases, giving rise to

(a) (b)

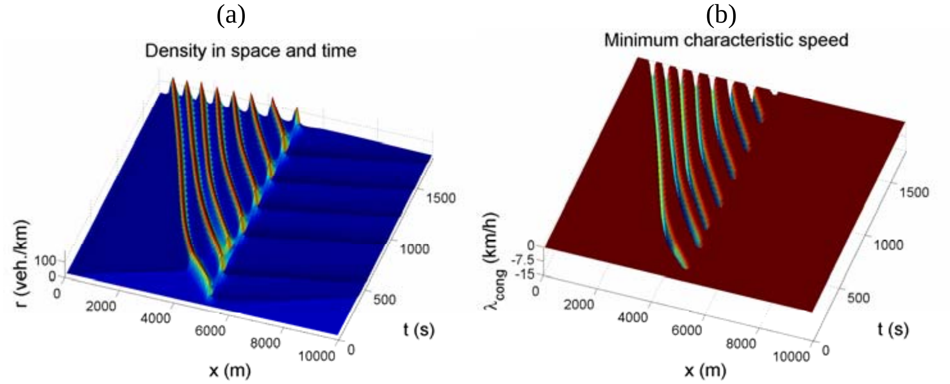

FIGURE 4: Unstable traffic flow with acceleration lane length = 50 m

the smaller right hand side of condition (21). This means that the main traffic is more unstable, which supports the conclusion that when approaching the end of the acceleration lane, drivers are willing to make more precarious decisions when merging (that is, accept smaller gaps when changing lanes), and, consequently, disturb the main traffic flow more significantly. The additional condition (21) explains why different congested traffic states are observed at different locations relative to the ramp entry.

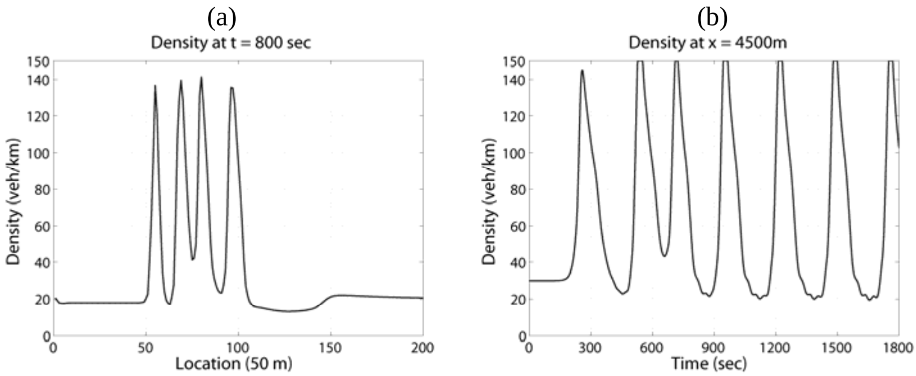

FIGURE 5: Oscillatory density at certain location and time

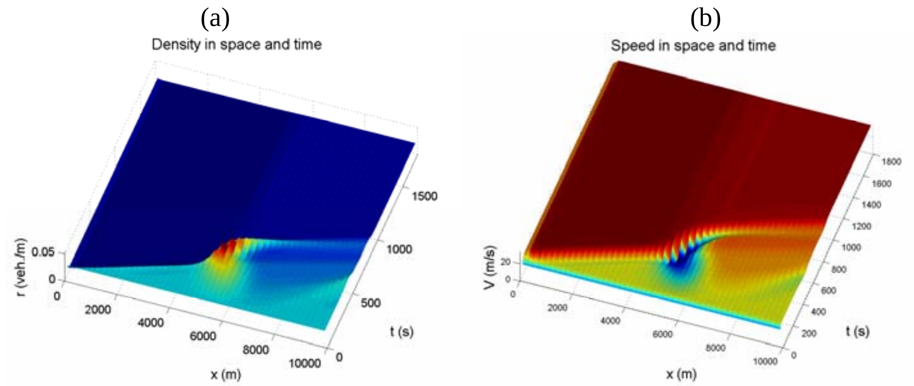

FIGURE 6: Stable traffic flow with acceleration lane length = 2000 m

5. CONCLUSIONS

In this paper, we have presented analytically and numerically the importance of the acceleration lane length on the stability of main traffic flow operations. The analysis has been carried out based on a linear stability method. According to these analytical findings, there are two conditions for the stability of the main traffic flow that need to be fulfilled in the presence of an on-ramp. Firstly, the stability condition for the initial condition of the main traffic must be satisfied. Secondly, the condition for ramp flow demand (amplitude of disturbances) and the acceleration lane length must be fulfilled as well. The additional condition regarding to the acceleration lane length explains why different congested traffic states are observed at different locations relative to the ramp. Simulation results of the presented model have supported our analytical findings. We have shown that different acceleration lane lengths, given the same perturbation of the on-ramp flow, result in various congested traffic states. The longer the acceleration lane length is the more stable the main traffic is.

This paper is itself not complete without case studies of real life applications. Further research should be put in the validation of the developed model based on a real traffic network. It is expected that the developed model is able to capture correctly the complex traffic phenomena at freeway discontinuities observed in literatures, for example by Sarvi et al. (2007).

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