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MODELLING STRATEGIC BEHAVIOUR IN ANTICIPATION OF CONGESTION

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Congestion in capacity-constrained urban environments is essentially the aggregate outcome of choices that individuals make in self-interest while assuming that others do the same. Existing models in transportation do typically no take the interaction and dependencies between different travellers into account. In this study, a new modelling approach was developed that incorporates an endogenous interaction term in a discrete choice model to describe the strategic choice behaviour of individuals. The approach accounts for different degrees of rationality, and choice situations, reflecting the extent of previous experiences. The choice models are linked to a model of how a control agent acts strategically, dependent on different objectives in optimizing usage of capacity-constrained facilities. The paper summarizes the approach, discusses the results of numerical simulations to assess its face validity and reports some main findings from two experiments that were conducted to empirically estimate the models.

KEYWORDS: Discrete choice, strategic behaviour, Nash equilibrium, quantal response, noisy introspection, Stackelberg game

1. INTRODUCTION

Assume you are approaching a highway junction. Modern information technology is recommending travellers for a particular destination to take a specific route. You are inclined to follow this advice, but then you realize that you may be better off and face less congestion if you do not comply and taking the alternative route because the recommended one may be congested if too many travellers comply with the recommendation. Then you realize that the situation is more complicated because other travellers may go through the same reasoning process. And what about the intentions of the information provider/control agent? Is the information provided in the interest of all travellers going to that destination or using the network, or are any benefits distributed across different types of travellers in a certain way? Depending on specific intentions, complying with the recommendation may or may not be your personal optimal choice. You realize that the information provider may also already have included this train of thought into his recommendation. So, what choice to make? If you have faced this situation before, you may reduce the uncertainty involved by basing your decision on previous experiences and on what you have learned in the past. If you have not been at that junction before, you can only mentally go over some iterative reasoning process of proactive and reactive behaviour of travellers and information providers in choosing a route.

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The situation described above typically arises because the effect of many individual choices leads to congestion or negative external effects at the aggregate level, which in turn may influence individual decision making. Under such circumstances, individuals may exhibit *strategic* behaviour in the sense that their choices are partly based on their conjectures of the choice behaviour of other individuals. Authorities trying to develop control strategies using newly developed information technologies have to predict travellers' strategic behaviour, not only with respect to the behaviour of others, but also with respect to how travellers respond strategically to the provided information.

Existing models used in transportation do typically not incorporate any mechanism to represent such strategic choice behaviour in anticipation of congestion. Activity-based models (see Timmermans et al., 2002 for an overview) and choice models in general assume that choice behaviour involves a value judgment (utility, satisfaction, attitude) and some choice mechanism to translate this value judgment into choice probabilities (e.g., Dellaert et al., 1996). The value judgment is typically a function of the attributes of the choice alternatives and distance or travel time separation only, conditional on socio-demographics. It does not take into account that the aggregation of such individual choices will lead to aggregate use patterns and perhaps congestion, which in turn may affect individual preferences. Models in transportation on the use of information to influence route choice, mode choice, departure time choice, destination choice, etc. do not differ much in this regard: information about congestion is typically defined as an given attribute of the transportation network and choices are modelled simply as an individual's response to this (and other) attributes (e.g., Ben-Akiva et al., 1991; Khattak et al., 1993; Chen and Mahmassani, 2004). Finally, traffic-flow models conceptualize congestion as a side constraint, not as an endogenous interdependent component. Driver behaviour is typically modelled in terms of a user optimum (i.e., minimizing travel time) with properties such as first-in first-out, causality and queuing theory (e.g., Yang and Meng, 1998; Tong and Wong, 2000; Lam and Huang, 2002; Szeto and Lo, 2004).

If indeed travellers act strategically (and information providers may do so as well), then the outcomes of current models will produce poor results and in principle can be improved by incorporating strategic feedback mechanisms in choice models. The PhD study summarized in this article was motivated by problems as described above (Han, 2006). The goal of the project is to develop and test classes of choice models for predicting strategic choice behaviour [5]. Methodologically, the proposed approach represents an integration of traditional discrete choice modelling and game theoretic interactive decision-making to capture the interactions and interdependencies both between individuals and between individuals and information providers. The aim of the study is not to develop a fully operational, specific model, for a specific application, but rather to explore and investigate particular classes of strategic choice models. The focus is on model types that potentially could be further elaborated and applied to many different problems of strategic choice behaviour under conditions of uncertainty.

The remainder of the paper is organized as follows. The next section discusses the model development. Section 3 describes results of numerical simulations that were conducted to illustrate the models and explore their performance. Various empirical

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[5] Note that such behaviour is not limited to transportation, but is relevant to all choice problems where emerging aggregate patterns impact individual preferences. In the paper, different application contexts were used in conducting the experiments to validate the models and different examples were given to illustrate and explain the model. In this article, we will use transportation examples only, referring to different transportation problems (route choice, departure choice, etc.) to suggest the variety of problems that can be addressed.

experiments in which individual subjects played against a computer, mimicking the strategic behaviour of other travellers and the information provider were conducted to estimate and validate the models. Some key results are briefly reported in Section 4. Finally, Section 5 gives a summary and discusses some possible future research directions.

2. MODEL DEVELOPMENT

2.1 Models of strategic choice behaviour between individuals

The interaction problem between individuals at the disaggregate level is formulated as a *N*-player *J*-option game and uses a discrete choice model with an endogenous interaction term to describe the strategic choice behaviour of a given individual, in which the choices of other individuals are explanatory variables in the utility function.

Assume a set of $N=\{1,2,...,n\}$ individuals, and for each individual $i \in N$ a strategy set $S_i = s_{i1}, s_{i2,}..., s_{iJ}$ of J_i pure strategic choices. $s=(s_1,\ldots,s_n)\in S$ is a strategy combination, and it consists of n strategies, one for each individual. $s_{-i}=(s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$ is a strategy combination of all individuals except *i*. For each $i \in N$, there is a *payoff function*, $u_i : S \to \Re$, where $S=\prod_{i\in N} S_i$ is the Cartesian product of the individual strategy spaces and representing choices of actions. This payoff function represents the utility that an individual derives from selecting a particular strategic response to the strategies that other individuals might choose.

Put in a discrete choice framework, individual *i*'s payoff for strategy *j* consists of three components: (1) a choice alternative specific component, $U^{\text{exogenous}}$, which expresses the exogenous attractiveness of a given alternative; (2) an interaction component, $U^{\text{endogenous}}$, which captures the expected impact of other individuals' choice behaviour; and (3) an idiosyncratic error term, ε, so-called individual "type", treated as an individual and alternative specific random variable whose distribution is common knowledge among all individuals, but whose exact value is private information to the individual. Thus,

$$U_{ij} = U_j^{\text{exougenous}} + U_j^{\text{endogenous}} + \varepsilon_{ij}\,, \tag{1}$$

$$U_j^{\text{exogenous}} = \boldsymbol{\beta}\mathbf{X}_j = \beta_1 X_j^{\text{preference}} + \beta_2 X_j^{\text{cost}}\,, \tag{2}$$

$$U^{\text{endogenous}} = h(f_j(n), n)\,. \tag{3}$$

The exogenous part accounts for the conventional external attributes, such as pure preferences and cost. The preference includes the variation in activity utility over time and space, and reflects the preference to become involved in the activity and the preferred time to conduct the activity. The cost includes time pressure and travel cost, which could, depending on the specific application, be further expanded to account for location accessibility, free flow travel time, route benefit, entrance fee, etc. (see also Gärling et al., 1998).

The endogenous part account for the impact generated internally through interdependent interaction. $f_j(n)$ allows a flexible specification of the effect on utilities due to inter-individual interaction, which depends on the characteristics of the context. An identical linear utility function implies that the marginal effects of additional other individuals choosing the same alternative are additively constant across different alternatives, while a more flexible nonlinear functional form allows the impact to increase or decrease at a varying rather than constant rate. Furthermore, one could define more specific distributed impacts of other individuals, such as a two-dimensional matrix of impact effects for alternative pairs of individuals. The main difference here is the number of individuals that choose each alternative is endogenously determined and not external.

2.1.1 Equilibrium conception: consistence condition

In equilibrium, we expect that individuals take into account the expected decisions of all other individuals when choosing their strategy. That is, in equilibrium, individuals choose strategies $(s_1^*(\varepsilon_1),\ldots s_i^*(\varepsilon_i),\ldots,s_n^*(\varepsilon_n))$ such that, for each individual *i* and each of its possible types ε_i , strategy $s_i^*(\varepsilon_i)$ maximizes its expected payoff across alternatives. Following traditional game theory (Fudenberg and Tirole, 1991), the Bayesian Nash equilibrium constitutes the optimal response that maximizes an individual's expected payoff, given his conjecture about other individuals' strategies. Each individual's pure-strategy Bayesian Nash equilibrium choice is formulated as:

$$s_i^*(\varepsilon_i) \in \arg\max_{S_i} \int_{\varepsilon_{-i}} U_i(s_i(\varepsilon_i), s_{-i}(\varepsilon_{-i}), (\varepsilon_i, \varepsilon_{-i})) f(\varepsilon_{-i}) d\varepsilon_{-i}\,, \tag{4}$$

where $U_i(s_i(\varepsilon_i), s_{-i}(\varepsilon_{-i})$ denotes the utility to individual *i* of choosing choice strategy $s_i(.)$ conditional on the individual's specific term ε_i , when the other individuals follow strategy $s_{-i}(.)$ and across the set of specific preferences ε_{-i} for all individuals, excluding individual *i*. Based on the common knowledge that the strategies of others are a function of their type, individual i could use his conditional beliefs $p(\varepsilon_{-i}|\varepsilon_i)$ to compute the expected utility of each choice and thus find his optimal response. This probability is calculated by integrating across the distribution of other individuals' error distributions. This function expresses the fact that the expected payoffs are calculated from the equilibrium distribution of joint strategies.

Due to the symmetry of individuals' error terms, which are draws from the same distribution, each individual will have the same conjecture about any one other individual *k*'s optimal strategy. That is,

$$\begin{aligned} p_{ij} &= \Pr(s_{kj}(.) = 1) = \Pr(E_k[U_{kj}(\varepsilon_{kj})] \geq E_k[U_{kj'}(\varepsilon_{kj'})]) \\ &= \Pr(E_k[\bar{U}_j(s_{-k})] + \varepsilon_{kj} \geq E_k[\bar{U}_{j'}(s_{-k})] + \varepsilon_{kj'}), \qquad \forall i \neq k, \forall j \neq j' \end{aligned} \tag{5}$$

Consequently, no individual's particular preferences affect the equilibrium. It implies that in equilibrium the probabilities for all individuals are identical $p_{ij}^* = p_{kj}^* = p_j^*$ as are the non-random utilities for each alternative $\bar{U}_{ij}^* = \bar{U}_{kj}^* = \bar{U}_j^*$. An individual's vector of conjectures over all alternatives is consequently defined by the set of *J* equations. This implies that, in the equilibrium, the belief probabilities that determine the expected payoffs on the right side of equation (5) match the decision probabilities on the left that result from applying a probabilistic choice rule to those expected payoffs.

By relaxing the assumption of perfect decision making (no error) and perfect foresight (no surprises), an alternative way to arrive at the strategic equilibrium is through probabilistic choice or "quantal response" (e.g., McFadden, 1981; McKelvey and Palfrey, 1995, 1998). When we take the Bayesian Nash equilibrium (BNE) as the way to transfer incomplete information to imperfect information that causes the imperfect decision, it is equivalent in the resulting mathematic equation to the quantal response equilibrium (QRE). There is a correspondence between QRE and BNE in the sense that a quantal response is a probability distribution of action profiles in a Bayesian Nash equilibrium.

2.1.2 The logit approach

The particular characteristics of the model depend on the distribution of the random component. A uniform distribution results in a linear model, while a normal distribution yields the probit model. Assuming individuals' types follow exponential distributions (IID) gives raise to the logit model, in which case the choice probabilities of individual *i* choosing alternative *j* are a proportional exponential function of expected payoff.

$$p_j^* = \frac{\exp(E[\bar{U}_j]/\mu)}{\sum_{j'=1}^{J} \exp(E[\bar{U}_{j'}]\mu)}. \tag{6}$$

This model can represent different decision styles dependent on the structure of the interaction between individuals and the degree of uncertainty. Let μ be a positive "error" parameter that reflects the uncertainty of individuals' beliefs about other individuals. Higher values of the error parameter make choice probabilities less sensitive to expected payoffs.

In the extreme case, as μ tends to zero, the variance of the utility disturbances approaches zero, and a deterministic choice model is obtained with no uncertainty about other individuals' utility. This means there is no error, and the model approaches the Nash equilibrium of complete information. The alternative with the highest expected payoff is selected with probability one, representing perfect rational behaviour. The commonly used Wardrop (1952) equilibrium of user optimum in traffic control and distribution are derived from this notion. The Nash equilibrium is a state of rest in the sense that no individual would want to change his own strategy unilaterally, knowing what strategies others are using. As μ trends to infinity, the variance of the disturbances approaches infinity. The probabilities are asymptotically the same for all alternatives, irrespective of the payoffs. That means that actions are based on error only, i.e. there is a unique equilibrium with equal probabilities for each alternative, reflecting perfect random behaviour. Extreme cases are useful for illustration. In reality, individuals are neither perfectly rational ($\mu = 0$) nor perfectly noisy ($\mu = \infty$), and it is the intermediate values of μ that are most relevant for explaining human behaviour. As such, the "error" parameter can be interpreted as a measure that allows us to model the bounded rationality of individuals with limited ability of anticipation and evaluating the extent to which actual behaviour is close to perfect rationality, represented by the Nash equilibrium.

2.1.3 Inconsistency: one-shot situations

The model described above is appropriate when individuals learned from repeated interactions what others could be expected to do. It may be less appropriate in new strategic situations. In such one-shot situations, individuals start with a lot of uncertainty about what others think, about what others think they think, and so on. In this introspection process, the belief distributions unlikely coincide with decision distributions. Following Goeree and Holt (2004), some noise can be injected into the model as follows:

$$p_{ij} = \Phi_{ij}^{\mu}\left(B^1\right) = \frac{\exp\left(\bar{U}_{ij}(B^1)/\mu\right)}{\sum_{j'=1}^{J_1}\exp\left(\bar{U}_{ij'}(B^1)/\mu\right)}, \tag{7}$$

where p_{ij} represents an individual's choice probability (B^0) , B^1 represents an individual's (first-order) belief about the other's choice, B^2 represents an individual's (second-order) belief about the other's belief about the individual's own choice, etc.

An individual's first-order belief is modelled as the other's logit best response, $B^1 = \Phi_{ij}^{\mu_1}(B^2)$, given the individual's second-order belief, B^2 . In other words, the process that produces an individual's first-order belief about what the other will do is modelled as the other's noisy best response given what the individual thinks the other thinks the individual will do. Thus, the higher-order beliefs follow logit best responses for ($n > k$) is:

$$\Phi_n^k = \Phi^{\mu_k} \circ \Phi^{\mu_{k+1}} \circ \ldots \circ \Phi^{\mu_n}, \tag{8}$$

which maps an individual's n-th order belief into his k-th order belief: $B^k = \Phi_n^k(B^n)$. Noise in this successive iteration increases to infinity as the number of iterations increases to $\mu_\infty = \infty$, because beliefs likely become more imprecise as the introspection processes evolves. The set of noisy rational responses is obtained by recursively applying the Φ^{μ} -mapping to P as the limit, using a higher error rate at every step:

$$B^0 = \lim_{n\to\infty} \Phi_n^0(p_0) = \lim_{n\to\infty} \Phi^{\mu_0} \circ \Phi^{\mu_1} \circ \ldots \circ \Phi^{\mu_k} \circ \Phi^{\mu_{k+1}} \circ \ldots \circ \Phi^{\mu_n}(p_0). \tag{9}$$

The probability p_0 can be chosen arbitrarily because the logit best response Φ^{μ} for $(\mu_\infty = \infty)$ maps any initial belief probability to a uniform probability (perfect randomness). That is, individuals' higher-order beliefs about what others think about what others think about … etc, become more and more diffuse and these (infinite) thought processes start out with uniform beliefs. The outcome of these thought processes (the individuals' decision choice probabilities) are not uniform. This is different from equilibrium concepts that map belief probabilities into actions that occur with the same probabilities.

Using a two parameter function to define the error rate during the iteration process, the model can accommodate a wide rang of scenarios:

$$\mu_k = t^k \mu_0, \tag{10}$$

where k is the iteration reasoning order. The parameter t determines how fast the noise increases with thinking iterations. When $t = 1$, the limit probabilities, p^*, must be invariant under the logit map: $\Phi^{\mu_0}(p^*) = p^*$. A fixed point of this type constitutes a logit equilibrium, and leads to a connection with our equilibrium expectation model as expressed in equation (6).

In the extreme, when t is very large, the model corresponds to one with all individuals involved in one-step reasoning and thinking that others make random choices. When

$t > 1$, the choice probabilities do not match the belief probabilities during the iterative thinking, the introspective process allows for surprises, which are likely to occur in scenarios that are unique and involve non-repeated interaction. Given the payoff parameters of the model, the introspective process in equations (7-9) predicts the probability with which an individual chooses strategies in one-shot situations. And this prediction will cover varying iterative reasoning processes about beliefs over beliefs, as reflected in different values of μ and *t*. Table 1 gives a summary.

TABLE 1: Overview of anticipation certainty in strategic situations

Indicator	Distribution	Expectation	Situations
$\mu \to \infty$	Equality	Random	Not applicable
$\mu_k = t^k \mu_0$	Noisy introspection	Imperfect	One-shot
$\mu = 6.6$	Quantal response	Imperfect	Repeated
$\mu = 0$	Nash equilibrium	Perfect	Ideal

2.2 Models of strategic choice behaviour between individuals and information providers

Besides the interaction between individuals, our approach further integrates the interaction between an information provider and individuals as a 2-player 2-stage game. In this game, one player represents the information provider, and the other represents the collective individuals. By formulating a game with a Stackelberg equilibrium, the information provider gets the leader's advantage without loosing the behavioural basis.

2.2.1 Interactive decisions between individuals and information providers

We assume that the information is reliable (i.e., easy to understand), although it may be "good" or "bad" in the sense that it may match or not match an individual's expectations. We assume that after receiving the recommendation, an individual will judge the recommendation on the basis of his/her own beliefs. If the recommendation is not in line with the individual's existing expectations, it is less likely that the individual will comply. Thus, this is a bi-level optimization problem, where the control agent tries to achieve his objective, while the individuals try to maximize their individual payoffs.

We assume that the control agent optimizes the offered information by, in advance, already taking into account the likely reactions of the travellers, implying that the problem is reduced to a single level optimization, known as a Stackelberg game. This can be defined and also be solved using the following fixed-point algorithm:

$$g^r = U_r(g^r, H^*(g^r)) \text{ subject to: } H^*(g^r) \text{ solves } H' = U_H(g^r, p^*), \qquad (11)$$

where $H^*(g^r)$ is the individuals reaction or best response to the information provider's strategy g^r. Mathematically, the Stackelberg optimum can be found by searching for the optimum value of one player's objective function along the optimum curve for the other player (e.g., Basar and Olsder, 1995). In theory, such an optimum is equal to or better than the Nash optimum, which reflects the points where both curves intersect.

2.2.2 Update of individual's expectation given information providers' recommendation

As we have indicated, individuals will have to update their expectations in light of the recommendations. Consider a timing problem under conditions of uncertainty. Each individual will have certain beliefs about the probability of other individuals choosing each time slot and their choice probabilities are derived from and coincide with these beliefs. Therefore, the collective traveller response is the weighted sum of self-expectation and recommendation. The updated expected probability for each time slot after having received the recommendation is expressed as follows:

$$h_j = (1-q)p_j^* + qg_j \;, \qquad (12)$$

where h_j is the updated probability for alternative j and used as input for the information provider to assess the objective achievement, and to revise the recommendation to optimize the expected outcome. The first term on the right side indicates the self-expected probability for alternative *j* and the second term represents the recommendation probability for the same alternative. Parameter *q* is the index of caution (versus boldness) and characterizes the degree to which the individuals are willing to follow the recommendation.

Two situations may arise: (1) the recommendation is consistent with the existing expectations, or (2) the recommendation is incongruent with the existing expectations. As noted in Arthur (1994), humans carry out localized deductions based on their current beliefs and act on them. They may be more conservative with respect to information that is far away from their concurrent belief, and more responsive in integrating information that is closer to their anticipation (Srinivasan and Mahmassani, 2000). Given these characteristics, the compliance rate of individuals is defined as a function of discrepancy between self-expectation and recommendation.

$$q = 1 - \frac{1}{2}\sqrt{\sum\nolimits_{j=1}^{J}(g_j - p_j^*)^2} \;. \qquad (13)$$

In the extreme case, as the recommendation (g_j) matches self-expectation (p_j^*), the recommendation is a confirmation of self-expectation and the compliance will be 100%. In the other extreme, when the recommendation is fully incongruent with self-expectation (e.g., $g_j = 0\%$ versus $p_j^* = 100\%$), the compliance is lower.

2.2.3 Information providers' objectives

Three typical roles of information providers are distinguished depending on their payoff function, leading to different equilibria. The information provider can act as a facility manager with the objective of equal usage across all time slots, and the objective function could be expressed in terms of the deviation from the reference points:

$$Max : U_r = \sum\nolimits_{j=1}^{J} X_j \qquad (14a)$$

subject to

$$X_j = \bar{x} - \alpha_1 \max\{N/J - Nh_j, 0\} - \alpha_2 \max\{Nh_j - N/J, 0\}, \qquad (14b)$$

where X_j is the facility utility of time unit *j*, $\bar{x}$ is the utility of the ideal situation, which in this case is reached when the number of visits is equal to the average, *N*/*J* , and which can be simply normalized to 0 as a reference. Nh_j is the number of individuals that will shown up at time *j*, given the recommendation. For simplicity, we assume that the facility utility function is linear in the number of individuals. With varying values between α_1 and α_2, it can capture the idea that a facility's surplus differs between negative deviations from the reference and positive deviations.

Alternatively, it can be a government agency with the objective of welfare maximization across all individuals, including the case that one individual's welfare is compensated by another's. This objective can be expressed as:

$$Max : U_r = \sum_{j=1}^{J} E[\bar{U}_j] Nh_j \ , \tag{15}$$

where $E[\bar{U}_j]$ is the average expected payoff for alternative time *j*, $j = 1, ..., J$, Nh_j is the number of individuals that will shown up at time *j*.

Yet another role is being a representative of collective individuals with the objective of equity, implying that utilities are the same for all individuals across all times (e.g., equal chance to get access at the most popular times). This equity objective will coincide with the user optimum in the sense that no single user can improve his payoff function by unilaterally changing his choice. Thus,

$$Min : U_r = \sum_{j=1}^{J} \left(E[\bar{U}_j] - \frac{1}{J} \sum_{k=1}^{J} E[\bar{U}_k] \right)^2 , \tag{16}$$

where $E[\bar{U}_j]$ is the average expected payoff for alternative time *j*, $j = 1, ..., J$.

3. NUMERICAL SIMULATION

The properties of the proposed model are tested through a series of numerical simulations. We start with examining the model of interactive decisions between individuals by testing sensitivities of three main components (see Han et al., 2004 for more detail). We continue with the model of interactive decisions between individuals and an information provider. The combined effects of two factors: one concerns testing three different levels of individuals' conjecture abilities, and the other concerns three different objectives that an information provider may have, are reported here. To illustrate the working of the model, it is applied to the problem of departure time choice in the context of a social activity.

The start time preference is the attractiveness of performing the activity during that time slot as a composite of exogenous influences. Because it is a social activity, some individuals may prefer a time slot when the probability of encountering many others is high, whereas other travellers may prefer to avoid such crowds. Thus, the utility function for time slots needs to include the expected numbers of individuals who will choose a particular time slot. For reasons of simplicity, we assume that the interaction component among individuals is a linear disutility in the number of individuals that have chosen the same time slot. Based on equation (1), the simplified systematic self-expected payoff function that is used in the illustration, $E[\bar{U}_2]$, is given thus by:

$$E[\bar{U}_2] = U_2^{\text{exogenous}} + \theta((N-1) p_2^* + 1) \ . \tag{17}$$

The equilibrium information provision and participation pattern is then determined by integrating the collective individuals' trade-off between self-expectation and recommendation into the information provider's objective function to search for the optimum.

The departure time choice under recommendation was simulated for 1000 individuals, given an assumed preference vector for each start time and the parameters of the model.

The specific settings are explained along with the algorithm. The following procedure was adopted in the simulations:

1. Draw 1000 sets of random values for the vector of preference from a normal density distribution with means (-3, 0, 3 for time slot 1, 2, 3 respectively) and a variance equal to 1. Every set defines an individual.
2. Use these values along with a marginal disutility of congestion (-0.006 per individual) to calculate the probability that an individual will choose a certain time slot, using equation (6). In this probability calculation, different values of μ are used to represent different levels of conjecture abilities: μ=1.0 to approximate the Nash equilibrium, $\mu = 6.6$ for the quantal response equilibrium, and $\mu_0 = 4.4$ and $t = 4.1$ for the noisy introspection model, truncated at 5 iterations.
3. Sum the probabilities for each start time across all 1000 simulated individuals, generating a distribution of choices for three start times without recommendation (scenario A).
4. Use these self-expected probabilities and the information provider's recommendation to deduce the updated probabilities using equation (12) with the compliance rate calculation using equation (13).
5. Sum the probabilities for each start time across all 1000 simulated individuals, generating a distribution of choices for three start times with recommendation (scenario B).
6. Use these numbers to assess to what extent the information provider's objective has been achieved using equation (14) for the equal usage objective (with $\alpha_1 = \alpha_2 = 1$, meaning that deviations from the reference level count the same for the information provider), equation (15) for the welfare maximum objective, and equation (16) for the user equity objective, and adjust the recommendation with a hill-climbing method.

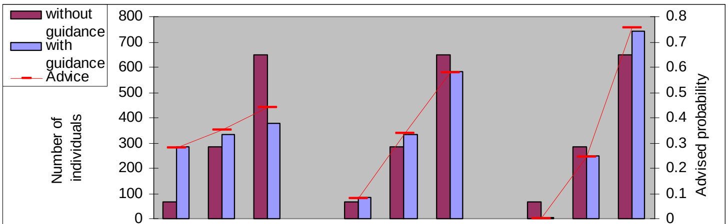
In order to compare the performance of the model for different objectives, the following measure was used. By replacing the compliance rate calculation in step 4 with all individuals completely following the recommendation (i.e., 100% compliance rate), an ideal distribution for each objective can be identified. With this ideal distribution, the information provider has full control and his objective is perfectly achieved. By measuring differences between the ideal distribution (NP_j^{ideal}) and the resulting distribution based on the behavioural model ($NP_j^{\text{behaviour}}$) for the same objective, a performance score for the information provider can be derived. This equation was specified as follows:

$$Y = \sum_{j=1}^{j=3} \frac{(NP_j^{\text{behaviour}} - NP_j^{\text{ideal}})^2}{NP_j^{\text{ideal}}} \,. \tag{18}$$

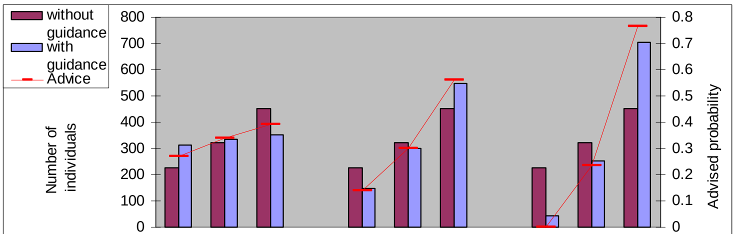
The smaller this score, the better the match. That is, the information provider's best performance is to obtain a score of 0, if possible. Thus, the differences between scores obtained for scenario A (no recommendation) and scores obtained for scenario B (with recommendation) reflect improvements in how well the information provider's objective is achieved by providing guidance to influence individuals' decision-making. Theoretically, the range of performance scores is different, and depends on the nature of the objective.

The results of the simulations are summarized in Figure 1. In general, it reveals that different objectives lead to differences in the generated advice and individual choices.

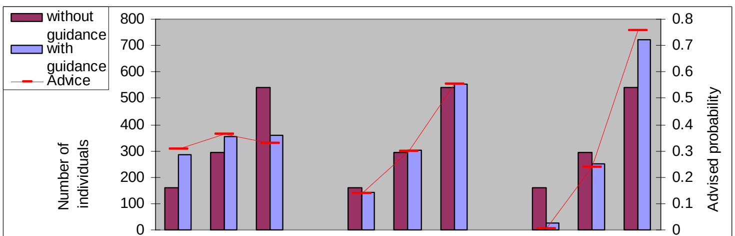
The consequent individual distribution with guidance shows obvious differences from the distribution without guidance, reflecting the apparent impact of information provision. From the results for the different objectives under the same level of conjecture ability among individuals, we can see that recommendations are relatively less effective in influencing individuals' decisions when the information provider intends to maximize welfare and achieve user equity. A possible reason for this difference in effectiveness is that recommendations are less influential when the information provider's objective is more or less the same as that of the individuals. With the recommendations being fairly similar to individuals' self-expectations, the outcomes will at best show subtle changes. In the case of pursuing equal usage of the facility over time, the individuals' objective is far removed from the information provider's objective and recommendations are therefore more influential. The extent to which the recommendation can differ from the individuals' self-expectations and still be effective, however, is limited due to potential shifts in individuals' compliance rates, and the consequent outcome is, therefore, always a compromise.



Nash equilibrium	Equal usage	Welfare maximum	User equity
without guidance	518.96	18.13	17.69
with guidance	12.97	0.01	0.05



Quantal response	Equal usage	Welfare maximum	User equity
without guidance	78.5	265.92	137.56
with guidance	2.66	55.11	2.98



Noisy introspection	Equal usage	Welfare maximum	User equity
without guidance	221.45	79.35	66.15
with guidance	10.43	44.12	0.98

FIGURE 1: Results for numerical simulations

In contrast, from the results for different levels of individuals' conjecture abilities with the same objective, we observed that different levels of conjecture abilities of individuals affect the achievement of the information provider's objective. Within the two equilibriums, the Nash equilibrium seems to perform better in both the welfare maximum and user equity case. But with equal usage level as the objective, the Nash equilibrium receives the lowest score. This difference is due to the assumption that in the Nash equilibrium the individual's best strategy is chosen with certainty, which implies the least willingness to adjust behaviour, especially when the objective underlying the recommendations is not in line with the interest of individuals. The results for the noisy introspection induced a good performance for the equal usage objective, but are worse for the welfare maximum objective. Compared with the quantal response equilibrium, the noisy introspection shows more improvement in case of the equal usage objective because individuals are easier to follow the recommendation when their reasoning between beliefs and actions is inconsistent. Generally, when individuals are less certain about what others will do, they show more willingness to follow the recommendations as in the noisy introspection situations, leading to a greater improvement in terms of the information provider's objective. Thus, the information provider may need to provide slightly different recommendations, and may need to assess the best possible achievement differently, if reality involves uncertainty about others' behaviour.

4. EXPERIMENTAL VALIDATION

In addition to a series of numerical simulation tests, two types of experiments were conducted to explore the properties of the proposed approach: whether individual strategically anticipates others choices and to what extend they react to it rationally. In general, conventional multinomial logit models (MNL) and random coefficient logit models (RCL) specified according to the different application context were estimated. Due to the space limitations, this section summarizes the experiment design, describes the estimation method if necessary, and discusses a few main findings.

4.1 Timing choice in one-shot situations

The first experiment used a paper and pencil questionnaire about "timing choice" to investigate congestion anticipation in one-shot situations. The experiment involves multiple tasks, covering two classes of activity: (1) A hypothetical choice of a daily compulsory activity with restrictive time preference (dinner time), and (2) A hypothetical and real-world choice for a discretionary activity on a less frequent basis with more flexible favourite timing (experiment participation time). Data for the study were collected in 2003-2004 in a number of sessions. Participants were first year undergraduate students, who participated for course credit[6]. A total of 118 participants registered and participated.

The objective of the compulsory activity "timing choice" was to investigate to what extent individuals take into account other individuals timing decisions when choosing their own time. In order to do so, two context versions were randomly assigned to each subject with three times (5:00 pm, 7:30 pm or 10:00 pm). Version A emphasised the

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[6] Students were selected partly for practical reasons (the second data collection involved playing an interactive computer experiment against a computer), and partly because this age cohort is likely most sensitive to strategic behaviour with respect to this context.

reason for waiting is driven by random variation, denoted " non-strategic choice" with the manipulated factors of expected waiting time (0, 10 or 20 minutes), and the probability of a 10 minutes additional delay (0%, 20% or 40% probability). Version B informed subjects that the reason for the unknown waiting time is a first come first serve basis. This was labelled "strategic choice" and the manipulated factor concerned capacity information ("close to capacity", "at full capacity", or "more than capacity").

The objective of the discretionary activity "timing choice" experiment was: (1) to investigate individuals' timing choices in a real-world setting involving potential congestion, (2) to compare the effect of the strategic anticipation of other individuals' behaviour as well as the non-strategic effects with the study of compulsory activity " timing choice", and (3) to obtain measures of individuals' non-strategic consumption timing choices in the same setting and to compare: (a) individuals' real-world timing choices, (b) their choices based on the noisy introspection model, (c) their choices based on a quantal response equilibrium, and (d) their choices based on a Nash equilibrium.

The three objectives were met by distinct parts of the experiment. To investigate the first objective, we used subjects' actual registration times for our experiment. Potential subjects for the experiment were recruited by registering through email so as to guarantee that they had no information about attendance levels per session except for the fact that congestion might occur. Three times were presented for each day in which subjects could participate in the experiment (2:30 pm, 4:00 pm or 5:30 pm). At the start of the experiment, subjects were also asked to report for each time slot the number of other students they expected would also participate and their anticipated waiting time. This data was later used as input in the timing distribution simulations and analysis. For the second and third objective, hypothetical choice tasks regarding subjects' timing preference for participating in an experiment were used with the same structure as "dinner time choice". Figure 2 gives a graphical representation of the intended plan.

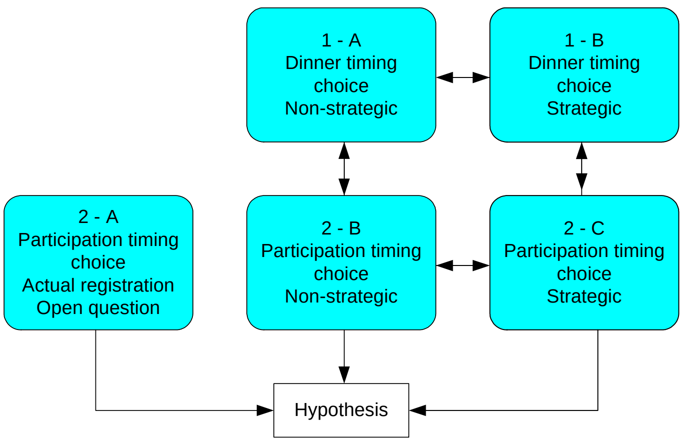

FIGURE 2: Structure of timing choice experimental design

The results of dinner timing choice show that 7:30 pm is the most popular time slot to conduct this activity, with a positive effect for information "close to capacity" in the strategic situation, and negative effects for waiting time and additional delay in the non-strategic situation. To test the interaction between timing preference and task version, we estimated an additional RCL model jointly across the two data sets as shown in Table 2. The interaction effect turned out to be significant, which suggests that individuals' timing preferences differ significantly depending on whether or not they need to take into account the timing choices of others (i.e., the effect of strategic considerations). They are able to anticipate other individuals' timing choices, and indeed adjust their own timing choices accordingly to avoid waiting due to possible congestion.

TABLE 2: Dinner timing choice model estimates (1-A & 1-B)[§]

	Strategic and non-strategic pooling effects		Version-time interaction effect	
Attribute	Parameter	t-value	Parameter	t-value
Intercept	4.98*	3.84	8.02 a	5.38
s.d.	3.47*	4.83	5.06 a	6.00
Time – 5:00 pm	0.53	1.06	-2.50 a	-3.70
s.d. time – 5:00 pm	3.16*	5.86	2.50 a	4.34
Time – 7:30 pm	3.96*	5.55	2.98 a	3.81
s.d. time – 7:30 pm	3.51*	6.15	0.49	1.13
Waiting time	-0.26*	-10.09	na	
Probability of additional wait	-0.07*	-8.71	na	
Information of close to capacity	1.30*	6.45	na	
Information of at full capacity	0.21	1.00	na	
Log likelihood null model		-964.86		
Log likelihood		-448.68		
Adjusted Rho-square		0.53		

[§] number of observations for "Strategic" is 183 (61 individuals); for "Non-strategic" is 513 (57 individuals); na = not applicable. * significant at $p < .05$ level. [a] Non-strategic estimate is significantly different from strategic choice task estimate at $p < .05$ level.

We also estimated a contrast parameter model across two classes of activities both in non-strategic situations and in strategic situations. The results confirm that individuals have similar timing preferences for an activity and dislike congestion, but the distribution depends on the nature of the activity. The shift in timing choices after receiving the specific information about the distribution of others proves that individuals adjust their strategic anticipation of congestion via information or experience about congestion.

The analysis of the participation timing data provides mixed support for the models (Figure 3). Based on a Chi-square analysis of the predicted distribution of demand relative to the actual distribution of demand, we find that the non-strategic timing choice model is the closest to the actual timing choices. Of the strategic models, both the quantal equilibrium model and the noisy introspection model differ significantly in terms of predicted numbers from the actual observed numbers of choices in each time slot, whereas the Nash equilibrium predicts a distribution that is considerably closer to the observed attendance levels, and not significantly different. Interestingly, subjects' self-reported expected number of other participants in each time slot is placed in between the projected quantal response equilibrium and the Nash equilibrium and does not differ significantly from the noisy introspection projections. In contrast, the self-reported expectations differ significantly from the actual distribution of attendance and from the non-strategic model projections.

Therefore, it provides evidence of the fact that in this real-world strategic choice situation individuals anticipate other individuals' timing choices. It shows that in this specific case individuals for some reason may not always take into account the congestion that they anticipate when making their own timing choices (i.e., they may not react to it strategically). There are a number of potential explanations such as the different certainty levels between own preference and others', the loose link between reality and idealism in attitudes, and so on.

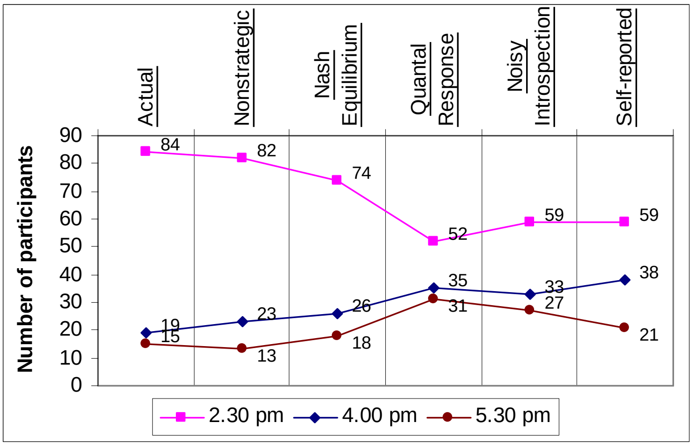

FIGURE 3: Participation timing choice comparison

4.2 Repeated strategic choice

To examine the effect of recommendation and learning in anticipation of congestion from one-shot to repeated situations in various scenarios, a second study on "strategic repeated choice" involving a controlled computer experiment with 32 tasks and 15 rounds each was conducted. Data for the study were collected in 2005 over four consecutive days. Participants were first year undergraduate students, who participated for course credit and a possible reward of ten euros based on their performance throughout the experiment. A total of 83 subjects participated.

In this experiment, we implemented our proposed model of equilibrium behaviour with a low error rate (μ = 1.0) to simulate the choice behaviour of others that the subject encountered in the experiment to guarantee that identical choice scenarios are replicated in multiple rounds and stimulate the real subjects to learn. The simulated choice behaviour has the property that when close to a Nash equilibrium the best strategy is chosen with a probability that is close to 1, reflecting the situation that the simulated players are perfectly rational with complete information about strategic interaction. The manipulated factors used for constructing different scenarios are: (1) the recommendation, (2) uncertainty of others' preference, (3) the compliance rate of others, and (4) the marginal disutility of others for congestion. The latter three factors controlled for subjects' possible state of affairs regarding others' strategic choices in anticipation of congestion.

The results show that individuals are able to anticipate other individuals' choices, and did indeed learn to adjust their choices accordingly to avoid congestion. The differences in choice frequencies confirm that individuals behave differently depending on whether there is recommendation, and the different objectives underlying the recommendation. With increased certainty, individuals stick more to their original preferred options. The results of a regression model and a contrast parameter model, using the data for the 1st round and the 13th round, showed that the effects of marginal disutility and compliance rate that others apply are significantly strengthened as individuals learn, while the effects of uncertainty of others' preference are not.

The choices of others anticipated by subjects when making strategic decisions were predicted, using the proposed strategic choice models as a starting point. Figure 4 illustrates some results. In general, it confirmed that in strategic situations, we should model choice behaviour differently: the model estimates with congestion prediction are significantly better compared with the model without prediction of congestion. The results of the strategic choice model for the 1st round and the 13th round data provided support for the notion that the effect of anticipated congestion differs significantly between no recommendation and with recommendation situations. When information is provided or a recommendation is given, individuals are more concerned with how to react to this recommendation.

One-shot situation strategic choice model estimates (1^{st} round)

Attribute	Without recommendation par.	dif	With recommendation par.
Option 1	0.47		-0.62*
s.d. option 1	1.19*		0.82*
Option 2	1.12*		0.56*
s.d. option 2	0.74*		0.80*
Base points	0.40*		0.57*
Anticipated congestion	-0.003	sig	-0.003
Recommendation 1			-0.36
Recommendation 2			-1.51*
Recommendation 3			-2.04*
Log likelihood null model	-364.74		-2553.18
Log likelihood	-319.09		-1636.23
Adj. McFadden R-sqr	0.12		0.36

Repeated situation strategic choice model estimates (13^{th} round)

Attribute	Without recommendation par.	dif	With recommendation par.
Option 1	0.41		-0.84*
s.d. option 1	0.31		0.24
Option 2	0.41*		0.28*
s.d. option 2	0.13		0.02
Base points	2.11*		1.15*
Anticipated congestion	-0.19*	sig	-0.07*
Recommendation 1			-0.62*
Recommendation 2			-3.19*
Recommendation 3			-4.20*
Log likelihood null model	-364.74		-2553.18
Log likelihood	-284.41		-1235.46
Adj. McFadden R-sqr	0.21		0.52

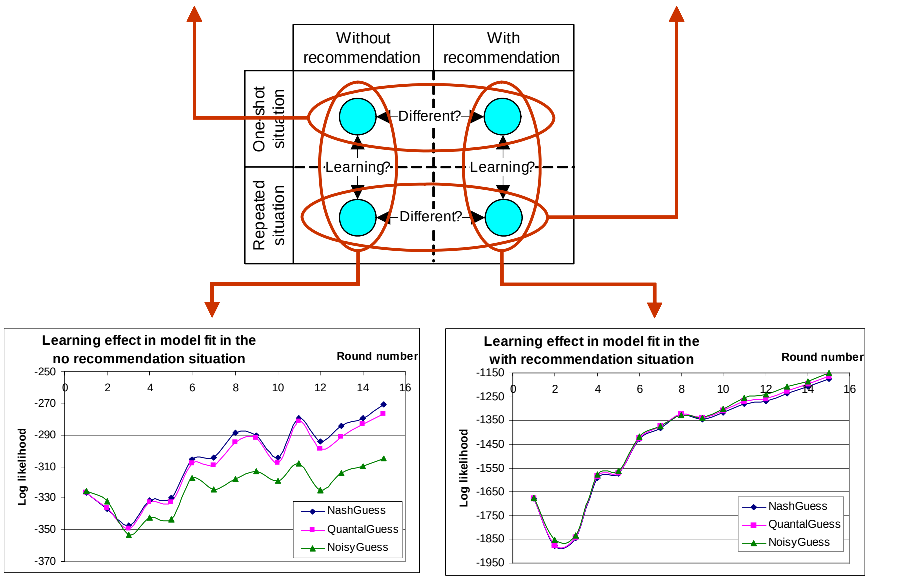

FIGURE 4: Illustration of repeated strategic choice results

In no recommendation situations, the estimates for the1st round data implied that the noisy introspection model, involving iterative reasoning with an increasing error rate, is most powerful in explaining individuals' strategic anticipation of congestion in one-shot situations. The underlying theory that the equilibrium model is more appropriate for predicting strategic choice when people have experienced the implications of their choices in repeated situations is confirmed by the reversal in the order of the best-worst fitting models, when moving from round 1 to round 15 with data pertaining to the no recommendation situation. Therefore, it confirmed that dependent on the situation, we should model strategic behaviour with different level of expectation certainty.

In situations with recommendation, there is a clear difference. Individuals may use recommendation as an indicator of other's choice behaviour, and, hence the needs to project an equilibrium outcome are softened. Therefore, the noisy introspection model has a higher explanatory power in terms of individuals' strategic anticipation not only for the first round data in one-shot situations, but also for later round data in repeated situations. Also, it can be concluded that the more complex the choice task, the slower the learning. When a recommendation is provided, a different marginal disutility and uncertainty about the preference of others are the factors that increase the complexity of strategic choices.

5. CONCLUSION AND DISCUSSION

In this article, we have briefly summarized the development and some numerical and experimental results of choice models that predict strategic choice behaviour under conditions of uncertainty where individual choices lead to emerging aggregate usage patterns which in turn influence individual choice behaviour. Three different models, describing different levels of previous experience and different degree of rationality were formulated. In addition, these models were linked to alternative actions of a control agent (information provider) that wishes to influence aggregate emerging usage patterns (optimum use) with varying objectives regarding the distributed effect for individual travellers.

The study has argued and provided both numerical and experimental evidence that many choice problems in transportation contain aspects of *strategic* choice in the sense that individual travellers anticipate the choice of other travellers while expect others do the same. This means that our current models, which typically do not contain any such strategic mechanisms, can and should be improved. Moreover, information providers wishing to optimize system use should take into account the extent to which individuals comply with recommendation while anticipating the extent to which other individuals will comply. The empirical studies show that the proposed approach effectively supports modelling these effects related to the interdependent nature of people's behaviour in a congested environment. By highlighting this interdependency, it extends traditional research from a focus on individuals' response to an exogenous environment to the role of endogenous interaction between different decision makers.

The proposed approach also allows one to determine optimal strategies of information provision. Three static stages of strategic behaviour, which are capable of predicting different steady-state distributions and how these distributions shift in response to changes in exogenous parameters, were proposed. Although the present model only considers the optimal start time of an activity under strategic recommendation, the principles underlying the model can be extended to model any time-space interaction between the supply side and demand side. This or a further elaborated version of the model might be especially valuable in situations involving control strategies, such as guidance systems in route choice or systems that help individuals plan their day-trip schedule, conditional on some city-wide objective. Most current systems either apply some algorithm with a weak behavioural basis to predict individuals' response to such information provision or use simple percentages of individuals complying with the recommendation. We argue that the inclusion of strategic interactive decision-making may represent an innovative way of predicting individuals' response and evaluating the impact of various policies. When strategic behaviour is important, it should lead to more precise and more sensitive forecasts of travel demand, and allow more effective control strategies.

It should be noted that the proposed approach is a general framework. It leaves open alternative specifications of the interaction components, and allows different decision styles: from perfect rational behaviour to completely random behaviour. By appropriately specifying the interaction term, the approach can be extended to model positive interactive effect as well as social dilemma problems that involved conflicts between self-interest and collective interest. In that sense, any application will require additional operational decisions, additional data collection and model estimation. Apart from that, the model itself can be further extended, for example, the current model does not explicitly address how inter-personal effects intertwine with personal constraints such as time budget, scheduling restrictions, and so on. To integrate resource limitations, the model can be further improved by: (1) using a nested logit structure; and (2) making an early distinction among individuals with respect to the type of constraints. To accommodate varying levels of rationality in the population across individuals, one may assume a multi-class population with different levels of expectation abilities. By introducing two scale parameters, one for strategically influenced variables (such as congestion) and another for external variables (such as preference), the model could account for different uncertainties. A more comprehensive model could integrate risk attitude to reflect framing effect of the recommendation (Kahneman, and Tversky, 1982). This approach has been taken in Han et al. (2005).

To model the dynamic process that evolves from one-shot noisy introspection to repeated quantal response equilibrium, an interactive learning approach that views equilibrium as the long-run outcome of a dynamic process of adjustment may be a promising approach. In strategic situations, individuals learn from changes in expectation based on (in)direct observations. That is, individuals not only learn and adjust their choice from their experiences with the chosen option, but also from the extent to which their expectations about the behaviour of others are consistent with actual choices (Han and Timmermans, 2006).

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