

## A NEW MULTI-CLASS CONTINUUM MODEL FOR TRAFFIC FLOW

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In this paper, we propose a new continuum model and study some qualitative properties. The new model contains an additional speed gradient term (anisotropic term) in comparison to Berg's model (Berg et al., 2000, *Physical Review E*, **61**, 1056-1066). This anisotropic term guarantees the property that the characteristic speeds can be made less than or equal to the macroscopic flow speed. We extend this model for multi-class traffic flow with heterogeneous drivers. Each user class is characterized by their choice of speeds in a traffic stream. The choice of the speed of a particular user class at any location on the highway is affected by the presence of all user class on that location. Numerical simulations show that the model is able to explain some of the observed traffic phenomena such as platoon dispersion that challenge old homogeneous models presented in the literature.

KEYWORDS: Continuum model, traffic flow, heterogeneous drivers, platoon dispersion

### 1. INTRODUCTION

Due to the great importance of efficient traffic for modern countries, the investigation of traffic flow problem is a very attentive topic of deliberation and researched for Engineers nowadays (e.g., Fukui et al., 2003; Bellomo et al., 2002) because it relates the adjustment, construction and management of traffic directly. Mathematical modeling of traffic flow has been a key tool to simulate or predict the behavior of transportation system. In the early stage of traffic engineering development, the methods to solve the traffic problems and design roads was largely empirical, built upon observations, measurements and statistical analyzes. Apart from the experimental observations, many physical models have been proposed (Gerlough and Huber, 1975; Helbing, 2001). An important question regarding to the applicabilities of these models lies in their capability to validate the empirical data collected by traffic flow studies to compare their models predictions with the real world situations. Each of these models has been tested either by simulation or against actual traffic flow measurements. Unfortunately, such validation processes has always invalidated the models (Wong and Wong, 2002). Measurements of traffic breakdowns on various freeways suggest that many dynamic aspects are universal and therefore accessible to a physical description.

To describe the dynamic properties of traffic on a homogeneous and unidirectional highway, Lighthill and Whitham, (1955) and Richards (1956) independently proposed a continuum model, which is known as the LWR model in the literature of traffic flow. In this model, a traffic stream model (relationship between fundamental variables: flow, speed and density) is supplemented by the continuity equation of vehicles and the resulting partial differential equation presumably could be solved to obtain the density as a function of space and time. LWR model, the simplest among all the continuum models exhibits a wide range of phenomena such as traffic "sound" waves, shocks and rarefaction waves.

However, the LWR model has its deficiencies, the most fatal one is that the speed is solely determined by the equilibrium speed density relationship. No fluctuation of speed

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around the equilibrium values is allowed, thus, the model is not suitable for the description of non-equilibrium situations like stop-and-go traffic etc.

In the past decades, many efforts were devoted to improving the LWR model through developing higher-order models, which use dynamic equation for the speed to replace the equilibrium relationship. Perhaps the most well known result of these efforts is the higher order model developed by Payne (1971). In the Payne model, the fluctuation of speed around the equilibrium values is allowed, thus, the model is suitable for the description of non-equilibrium situations like stop-and-go traffic etc. Later several authors (e.g., Phillips, 1979; Kerner and Konhäuser, 1993; Zhang, 1998) have suggested a considerable number of modifications to Payne's model.

Nevertheless, there are still some puzzling traffic phenomena observed on the highway, such as the two-capacity or reverse- state in the fundamental diagram, hysteresis of traffic flow and platoon dispersion that these proposed models can not explain. A second type of extension to the LWR model segregates traffic flow further. The freeway to be modeled can be divided into traffic lanes as was done by Munjal and Pipes (1971) and Daganzo (1997); user-classes that have no effect on the flow of traffic can be identified (e.g., Daganzo, 1995); and several vehicle types can be distinguished as in Wong and Wong (2002). The new behavioral theory of Daganzo (2002) also fits in this category of extensions.

Recently, multi-class continuum models have been developed in an attempt to explain these puzzling phenomena by modeling user lane changing behavior and/or multiple vehicles type. The introduction of heterogeneous vehicle properties and drivers behaviors has already been applied in other types of models. Hoogendoorn and Bovy (1996) proposed a semi discrete (i.e. continuous in time and discrete in space) multi-class generalization of the Payne model. Hoogendoorn and Bovy (2000) also formulated a multi-class model based upon gas kinetic principles. The multi-class concept has also been employed in Wong and Wong (2002), which was extended from the LWR model. Cho and Lo (2002) introduced a systematic self-consistent multi-class multilane traffic model, which is derived from the vehicular Boltzmann equation and the traffic dispersion model. Helbing (1997) presented a tentative gas-kinetic model for multilane traffic flow operations. Mason and Woods (1997) studied several aspects of the behavior of a multispecies car-following model both on circular and on straight roads.

Heterogeneous dynamic traffic flow models are extremely useful for intelligent transportation system's applications. The incorporation of these heterogeneous properties within a macroscopic model leads to a fast tool that can be used in real life applications. Although these models represent some improvements over unidirectional and homogeneous drivers models in some aspects, they do not resolve the wrong way travel problem that exists in almost all the previous higher order models. To overcome this difficulty, Aw and Rascle (2000), Jiang et al. (2002) and Zhang (2002) proposed a second class of higher order models. Aw and Rascle (2002) developed a model to suppress the gas like behavior by replacing the space derivative of density with a convective derivative. Both Jiang et al. (2002) and Zhang (2002) derived a macroscopic equation with anisotropy from car following models. These models are different from the first class of models in the sense that the anticipation term is a speed gradient term instead of a density gradient term. Thus, these models are referred to as speed gradient (SG) models in the literature of traffic flow.

Shu (1997) extensively studied the essentially non-oscillatory (ENO) and weighted essentially non-oscillatory schemes (WENO) for hyperbolic conservation laws. Zhang et al. (2006) developed a weighted essentially non-oscillatory numerical scheme for a

multi-class traffic flow model on an inhomogeneous highway but the WENO type scheme developed by Zhang et al. (2006) is applicable only on the system of hyperbolic conservation laws.

In this paper, we attempt to make a multi-class anisotropic continuum model to incorporate a distribution of heterogeneous road users. To test the effectiveness of the proposed model, we perform numerical experiments for two different test cases.

In Section 2, we present an improved continuum model recently developed by the authors (Gupta and Katiyar, 2005). The new model is based upon the car following model given by Jiang et al. (2002) and using the series expansion between headway and density given by Berg et al. (2000). In Section 3, we analyze some qualitative features of the model. We then develop a multi-class model by extending our anisotropic continuum model with heterogeneous drivers in Section 4. Next in Section 5, we shortly discuss the numerical scheme and investigate the model using macroscopic simulation for the test cases. Finally in Section 6, we provide a discussion on our results.

## 2. A NEW ANISOTROPIC CONTINUUM MODEL

The car following model was developed to model the motion of vehicles following each other on a single lane without overtaking. Car following models present the only class of models that describes each vehicle in a deterministic manner including the response to local variables such as speed, headway and change of headway. Therefore they seem to be of great importance with regards to autonomous cruise control systems, which should stabilize the flow as well as maximize the flow rate. In this paper, we follow the model in Jiang et al. (2002) of road traffic, in which the acceleration of every vehicle is determined by

$$\frac{dv_n(t)}{dt} = a(V_e(b_n) - v_n(t)) + \alpha\lambda\Delta v, \quad (1)$$

where  $\Delta v = v_{n-1}(t) - v_n(t)$ , with  $v_{n-1}$  and  $v_n$  being the speed of leading and following car respectively,  $V_e$  is the optimal velocity function,  $b_n$  is the headway and  $a$  is the driver's sensitivity which equals the inverse of the driver's reaction time.

For non-homogeneous conditions, the headway can be written as a perturbation series (Berg et al., 2000). Inserting the approximate expression for headway into equation (1), we can easily obtain the traffic flow dynamics equation (Gupta and Katiyar, 2006). The full description of this new non-equilibrium theory is given by a system of partial differential equations with the first being the vehicle conservation and the second speed dynamics.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \quad (2)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = a(\bar{V}(\rho) - v) + a\bar{V}'(\rho) \left( \frac{1}{2\rho} \frac{\partial \rho}{\partial x} + \frac{1}{6\rho^2} \frac{\partial^2 \rho}{\partial x^2} - \frac{1}{2\rho^3} \left( \frac{\partial \rho}{\partial x} \right)^2 \right) - 2\beta c(\rho) \frac{\partial v}{\partial x}, \quad (3)$$

where  $\bar{V}'(\rho) = d\bar{V}(\rho)/d\rho$ ,  $\beta$  is a non-negative dimensionless parameter and  $c(\rho) < 0$  is the traffic information speed given by

$$c^2(\rho) = -\frac{a\bar{V}'(\rho)}{2}. \quad (4)$$

Note that if  $\beta = 0$ , then the model reduces to the model in Berg et al. (2000) and upon dropping the term of  $(\partial\rho/\partial x)^2$  the model converts to the model given by Zhou et al. (2002). Equation (3) is analogous to the model in Zhang (2003). However, an important difference between that model and the new model lies in the coefficient of higher order terms.

Lee et al. (2001) proposed a new way of obtaining macroscopic models from microscopic models, via a coarse-graining procedure and allow us to carry out simultaneous micro-and macro- simulations of neighboring freeway sections.

### 3. QUALITATIVE PROPERTIES OF THE MODEL

The system (2) and (3) has a similar structure to the models in Berg et al. (2000) and Zhou et al. (2002) but is more general than these models. On comparing, we get that our model has an additional term  $2\beta c(\rho)\partial v/\partial x$ . Moreover the term  $c(\rho)$  varies with optimal velocity function. These differences, however, are not the structural differences. Hence, one would expect that the new model behaves roughly the same as Berg et al.'s model and Zhou et al.'s model and perhaps gives a more accurate description of traffic flow owing to its greater generality.

We rewrite the system (2) and (3) as follows:

$$\begin{Bmatrix} \rho \\ v \end{Bmatrix}_t + \begin{bmatrix} -\frac{v}{2\rho} & \rho \\ -a\bar{V}'(\rho) & v + 2\beta c(\rho) \end{bmatrix} \begin{Bmatrix} \rho \\ v \end{Bmatrix}_x = \left\{ a(\bar{V}(\rho) - v) + a\bar{V}' \left( \frac{0}{6\rho^2} \frac{\partial^2 \rho}{\partial x^2} - \frac{1}{2\rho^3} \left( \frac{\partial \rho}{\partial x} \right)^2 \right) \right\}. \quad (5)$$

The corresponding homogeneous inviscid system of (5) can be written as

$$\begin{Bmatrix} \rho \\ v \end{Bmatrix}_t + \begin{bmatrix} -\frac{v}{2\rho} & \rho \\ -a\bar{V}'(\rho) & v + 2\beta c(\rho) \end{bmatrix} \begin{Bmatrix} \rho \\ v \end{Bmatrix}_x = 0, \quad (6)$$

or  $\frac{\partial U}{\partial t} + [A] \frac{\partial U}{\partial x} = 0$ , is strictly hyperbolic, with eigenvalues  $\lambda_1 < \lambda_2$ .

$$\lambda_1 = v + \left( \beta + \sqrt{1 + \beta^2} \right) c(\rho), \quad (7)$$

$$\lambda_2 = v + \left( \beta - \sqrt{1 + \beta^2} \right) c(\rho), \quad (8)$$

and right eigenvectors

$$r_1(\rho, v) = \left( 1, \left( \beta + \sqrt{1 + \beta^2} \right) \frac{c(\rho)}{\rho} \right)^t, \quad (9)$$

$$r_2(\rho, v) = \left( 1, \left( \beta - \sqrt{1 + \beta^2} \right) \frac{c(\rho)}{\rho} \right)^t. \quad (10)$$

Moreover, the two characteristic fields of (6) corresponding to the right eigenvectors are genuinely nonlinear:

$$\nabla \lambda_1(\rho, v) r_1(\rho, v) = \left( \beta + \sqrt{1 + \beta^2} \right) \left( c'(\rho) + \frac{c(\rho)}{\rho} \right) < 0, \quad (11)$$

$$\nabla\lambda_2(\rho, v)r_2(\rho, v) = \left(\beta - \sqrt{1 + \beta^2}\right) \left(c'(\rho) + \frac{c(\rho)}{\rho}\right) > 0, \quad (12)$$

where  $\nabla\lambda_{1,2}$  are the gradients of  $\lambda_{1,2}$  with respect to  $\rho$  and  $v$ .

The homogeneous system (6) has two families of traffic sound waves, shock and rarefaction waves, one family for each characteristic field. For the first characteristic field the properties of these waves are quantitatively identical to those of the LWR model because of  $\lambda_1 \leq v$ . However, for the second characteristic, the waves behaves quite differently as they travel faster than traffic ( $\lambda_2 \geq v$  and  $\nabla\lambda_2(\rho, v)r_2(\rho, v) > 0$ ). This means that the future conditions of the traffic flow will be affected by the traffic conditions behind the flow. Such type of behavior, however, can be controlled by the factor  $\beta$  in our model. We call it the anisotropic factor. Note that  $\beta \gg 1$ , the second characteristic approaches  $v$ , the velocity of the traffic. Thus, information can never reach vehicles from behind. The term having this anisotropic factor in the system (5) is known as anisotropic term. In our general homogeneous model, the anisotropic nature of the model does not depend directly on the viscosity term.

It is difficult to write our system in a conservative form because in the development of equation (3) i.e. equation of motion, no conservation law is used. For dealing of the shock wave or discontinuity, the traveling wave solution is already examined and analyzed analytically as well as numerically (Gupta and Katiyar, 2005).

#### 4. EXTENDED CONTINUUM MODEL

In this section, we formulate a new model for heterogeneous drivers from the new anisotropic continuum model. Let there be  $M$  classes of users with different behavior, described by different conservation laws. Each class has different speed choice behaviors in response to the same density on a highway section. It means that for a given total density, there exists a distribution of speeds by different user classes. It is assumed that as density increases, the variation around mean speed decreases. Total density on a highway section is given by

$$\rho(x, t) = \sum_{m=1}^M \rho_m(x, t), \quad (13)$$

where  $\rho_m(x, t)$  is the density of user class  $m$  in the space time domain.

The fundamental variable of traffic flow (i.e. velocity, density and flow rate) of a particular class are related as

$$q_m(x, t) = v_m(x, t)\rho_m(x, t), \quad \forall m = 1, 2, 3, \dots, M. \quad (14)$$

According to different driving behavior, each user class should satisfy the following law of conservation of vehicles.

$$\frac{\partial \rho_m(x, t)}{\partial t} + \frac{\partial q_m(x, t)}{\partial x} = 0, \quad \forall m = 1, 2, 3, \dots, M. \quad (15)$$

It states that the density changes according to the balance between the inflow and outflow of vehicles of user class  $m$  along a topographically homogeneous highway section. The velocity dynamics equation for multi-class user is given by

$$\begin{aligned} \frac{\partial v_m}{\partial t} + v_m \frac{\partial v_m}{\partial x} &= \frac{\bar{V}_m(\rho) - v_m}{\tau} \\ &+ a\bar{V}'_m(\rho) \left( \frac{1}{2\rho_m} \frac{\partial \rho_m}{\partial x} + \frac{1}{6\rho_m^2} \frac{\partial^2 \rho_m}{\partial x^2} - \frac{1}{2\rho_m^3} \left( \frac{\partial \rho_m}{\partial x} \right)^2 \right) \\ &- 2\beta c(\rho) \frac{\partial v_m}{\partial x}, \quad \forall m = 1, 2, 3, \dots, M. \end{aligned} \quad (16)$$

The above equation shows the changes in the traffic conditions experienced by an observer moving along with the vehicles of class  $m$ . Here  $\bar{V}_m(\rho)$  is the equilibrium velocity for  $m$ th user class, which depends on the free flowing speed of the user class  $m$  and also on the total density at any location at any time. It means that the choice of speed of a particular user class is affected by the presence of this user class, but also by all other user classes on the highway. Note that the multi-class model (15 and 16) reduces to the original anisotropic continuum model for homogeneous driver described in Section 2.

## 5. NUMERICAL SCHEME AND COMPUTATIONAL EXPERIMENTS

### 5.1 Solution procedure

In this section, we present the numerical method of the multi-class new anisotropic model using the first order Lax-Friedrichs scheme. The developed model can not be written in the conservative form. Hence, the higher order WENO type scheme for multi class traffic flow models recently developed by Zhang et al. (2006) is not applicable on the developed model. It is still remains to develop the higher order WENO type scheme for such type of higher order models. The freeway to be modeled is divided into  $I$  nodes and the period of analysis in to  $J$  time steps. The following difference equations are obtained by applying the Lax-Friedrichs scheme on the set of partial differential equations (14, 15).

$$\rho_m(i, j+1) = \frac{1}{2}(\rho_m(i+1, j) + \rho_m(i-1, j)) - \frac{\Delta t}{2\Delta x}(q_m(i+1, j) - q_m(i-1, j)), \quad (17)$$

$$\begin{aligned} v_m(i, j+1) &= \frac{1}{2}(v_m(i+1, j) + v_m(i-1, j)) \\ &- \frac{\Delta t}{2\Delta x} \left( 2\beta c + \frac{1}{2}(v_m(i+1, j) + v_m(i-1, j)) \right) (v_m(i+1, j) - v_m(i-1, j)) \\ &+ a\Delta t \left( \bar{V} - \frac{1}{2}(v_m(i+1, j) + v_m(i-1, j)) \right) \\ &+ a\Delta t \bar{V}' \left( \frac{(\rho_m(i+1, j) - \rho_m(i-1, j))}{2\Delta x(\rho_m(i+1, j) + \rho_m(i-1, j))} \right. \\ &\quad \left. + \frac{(\rho_m(i+1, j) - 2\rho_m(i, j) + \rho_m(i-1, j))}{6(\Delta x)^2(\rho_m(i+1, j) + \rho_m(i-1, j))^2} \right), \end{aligned} \quad (18)$$

where index  $i$  represents the road section and index  $j$  represents time. Here  $\Delta t$  and  $\Delta x$  are the grid sizes of the finite difference mesh in time and space dimensions, respectively.

These equations give the density and velocity value of the  $m$ th class at the  $i$ th node at time step  $j+1$  from the known value obtained at the previous time step  $j$ . Given the initial and boundary conditions, we can easily calculate the velocity and density profiles of different classes at any time using equations (17) and (18).

## 5.2 Description of experiments

To investigate whether the new multi-class anisotropic model can be used effectively under the important traffic conditions; we conducted numerical experiments using two hypothetical test cases. Case I describes a topographically homogeneous highway of a length of 32.2 km and for the case II, we consider a ring road of 32.2 km length. For equilibrium speed density relationship, we use the following isotropic form (modified from the model in Drake et al. (1967) for homogeneous users) for all user classes.

$$\bar{V}_m(\rho) = U_m(\rho_1, \rho_2, \dots, \rho_m) = u_{fm} \exp\left(-\frac{\rho^2}{2\rho_0^2}\right), \quad (19)$$

where  $u_{fm}$  is the free flowing speed of the user class  $m$  on an empty road,  $\rho$  is the total density defined in equation (13) and  $\rho_0$  is the common parameter, which is constant for all user classes. Each class is characterized by its free flowing speed  $u_{fm}$ .

We identify nine different classes characterized by the free flowing speed of 15.0, 19.0, 23.0, 27.0, 31.0, 35.0, 39.0, 43.0, 47.0 m/sec in the experiments. The distribution of these user classes on the experimental highway is shown in Figure 1. The maximum density or the jam density for all the classes is taken as 0.2 veh/m. Figures 2 and 3 show the speed density and flow density plots for all classes of users. It is clear from Figure 3 that the flow increases firstly and after a critical value decreases. At a critical density value, the traffic velocity reaches its maximum. It can also be seen that the maximum flow corresponding to the faster users is higher than the slower ones, and for a given total density there is a distribution of speeds associated with different users with diminishing variance, when density increases. For numerical calculations, the test road section is divided into equal intervals of length zone, and the time grid size is taken as 2.0 sec.

## 5.3 Experiment 1

In this experiment, we consider a homogeneous highway with a length of 32.2 km. Due to a certain incident on the highway, it is assumed that the initial density distribution platoon with a peak of 0.14 veh/m is formed as shown in Figure 4. The left boundary has no inflow for all time, and the right boundary is a free outflow (Neumann boundary condition). We assume an equal distribution of drivers in the platoon. The solution of this problem can be worked out using the finite difference method outlined in section 3. The related parameters of our model are as follows:  $\beta = 20.0, T = 14 \text{ sec}$ .

The results from the analysis are shown in Figures 5 and 6. Figure 5 depicts the density contour plot that develops from the initial density distribution. Figure 6 shows the density profile of the platoon along the highway at different times. It is clear from the figures that the front and end of the platoon smoothed over time. As already pointed out by Berg et al. (2000), the discontinuity cannot move with the initial shape in the real traffic flow situations. There is experimental evidence that platoon decrement in real traffic flow situation. This can be easily explained in terms of heterogeneous driver behavior. In a platoon, a vehicle having a faster desired free speed will move more quickly towards the front of the platoon than the slower vehicle (having the less desired speed). Due to such a type of dispersion of the platoon, a slower vehicle will remain at the tail.

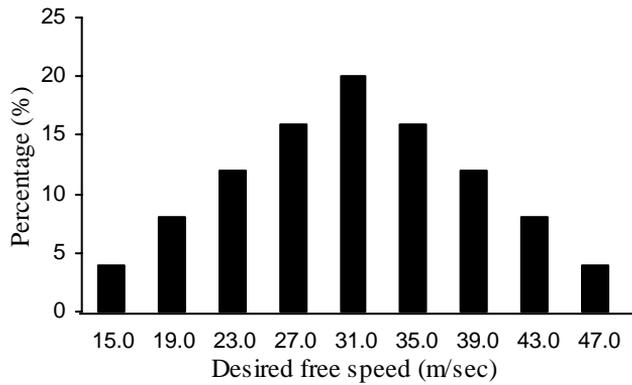


FIGURE 1: Distribution of drivers in the platoon

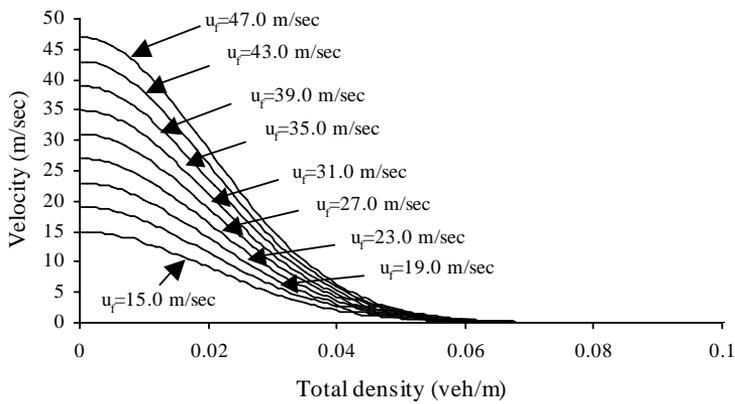


FIGURE 2: Speed density curves for different user classes

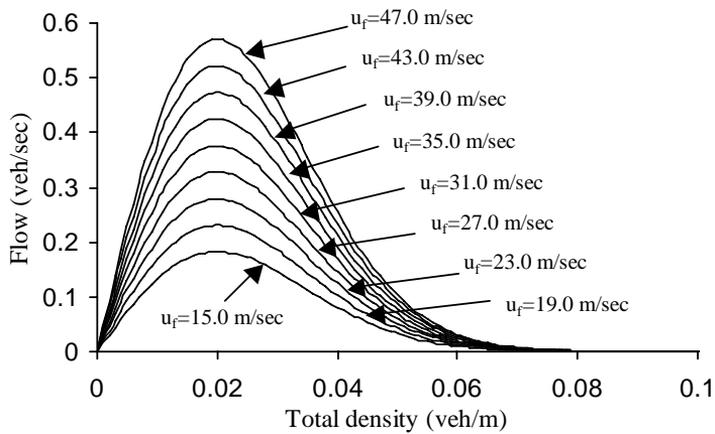


FIGURE 3: Flow density curves for different user classes

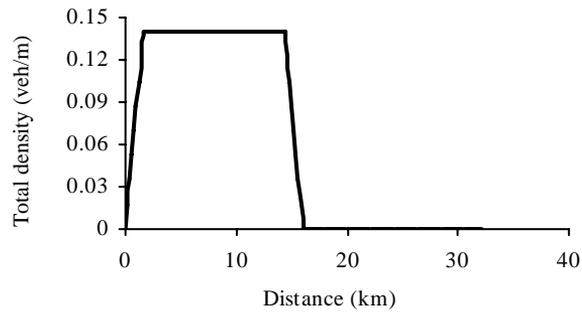


FIGURE 4: Initial platoon on the highway for Experiment 1

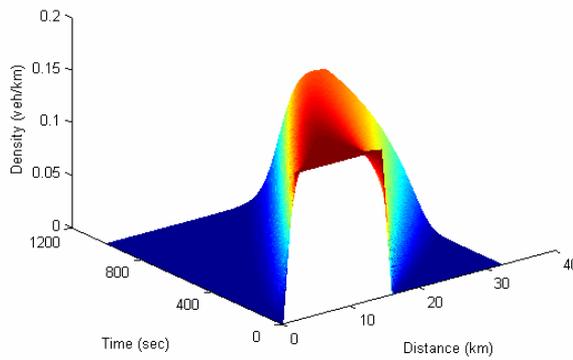


FIGURE 5: Three-dimensional density plot for Experiment 1

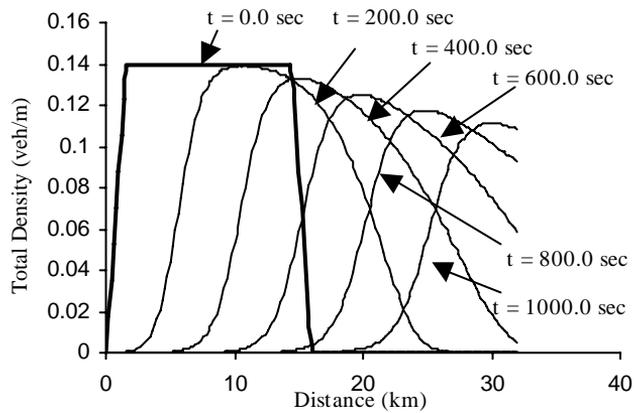


FIGURE 6: Density profile at different times for Experiment 1

By taking different initial density distribution, we can remark that the shape of the diffused platoon is solely depend upon the distribution of the desired free speeds. It is clear from the Figures 5 and 6 that the new multi-class anisotropic model provides predictions of the diffusion behavior of the vehicles in a platoon.

#### 5.4 Experiment 2

In this section, we will illustrate macroscopic multi-class simulation using a hypothetical ring road of 32.2 km long. In order to show the capability of the multi-class model, presented in this paper in describing platoon dispersion properly, we consider three platoons of vehicles of different densities at different locations initially formed on the ring road. The three regions of different densities 0.04, 0.08 and 0.12 are respectively located at  $X_1 = (3\text{ km}, 6\text{ km})$ ,  $X_2 = (9\text{ km}, 12\text{ km})$  and  $X_3 = (15\text{ km}, 18\text{ km})$ . We consider the same free speed and equilibrium velocity as in Experiment 1. The solution procedure is applied again to solve the problem and results are shown in Figures 7 and 8.

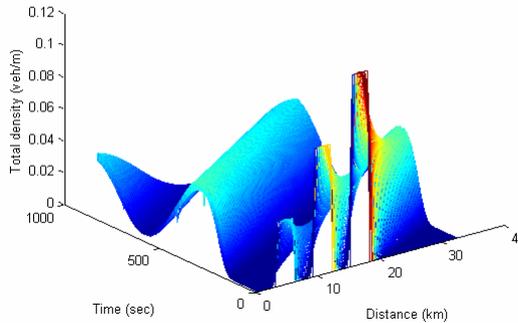


FIGURE 7: Three-dimensional density plot for Experiment 2

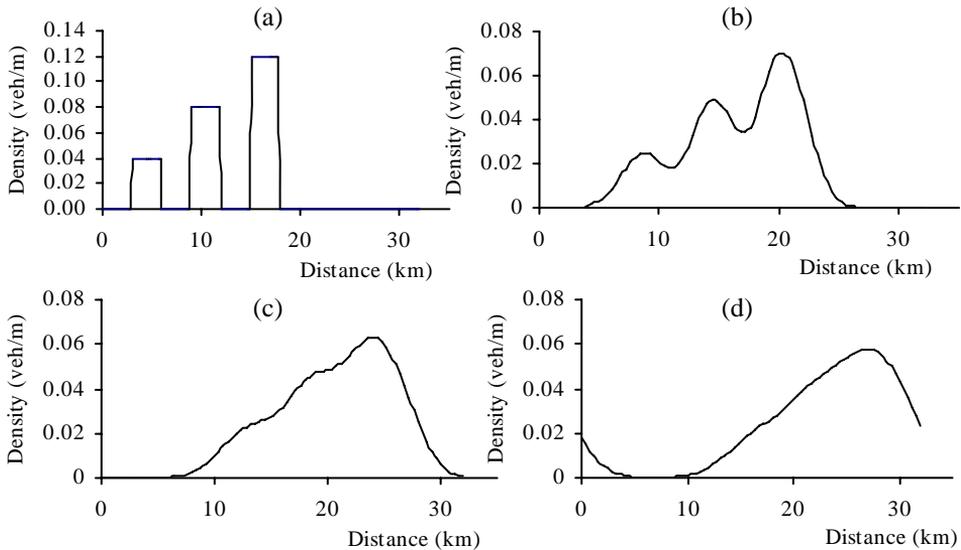


FIGURE 8: Evolution of traffic density on a ring road of circumference 32.2 km for (a)  $t = 0.0\text{ sec}$  ; (b)  $t = 140\text{ sec}$  ; (c)  $t = 280\text{ sec}$  ; (d)  $t = 400\text{ sec}$  .

Figure 7 shows the three-dimensional plot of the density variation with respect to space and time. Figure 8 illustrates the density profile of the platoons on the ring road at different times. It is clear from the figure that the new model provides correct predictions of the road traffic under the given initial condition. Out of the three platoons in  $X_1$ ,  $X_2$ , and  $X_3$  regions, the velocity in the  $X_1$  region is much higher than the velocity in the  $X_3$  region. Figure 8b describes that the smaller platoon smoothed quickly in comparison to the bigger platoon, which shows the correct picture of the highway traffic. After some time all the platoons merges into a single platoon having a definite shape. It can easily be shown from the figure that as time increases, maximum value of density within the platoon decreases.

Therefore for both the experiments, the above result shows a good agreement to the results found by Wong and Wong (2002) and Herrmann and Kerner (1998).

## 6. CONCLUSION

In this paper, we presented a multi-class generalization of a new anisotropic continuum traffic flow model. The new single class traffic flow model is derived from the car following model and practically includes some of the well-known non-equilibrium models as a special cases. However, an important difference between the new model and the previously developed model is that our model is isotropic. But the isotropic behavior can be controlled by the anisotropic factor  $\beta$  in the limit  $\beta \rightarrow \infty$ , the isotropic behavior dies out and the model becomes fully anisotropic. Our model is analogous to the model in Kerner-Konhäuser (1993). However, an important difference between the Kerner-Konhäuser model and the new model lies in the coefficient of the higher order terms. In the Kerner-Konhäuser model the coefficients are assumed to be constant but in our model they actually depend on density ( $\rho$ ). We proposed a simple finite difference scheme to carry out the numerical simulation and discuss the applicability of the scheme in some special cases. Finally the numerical tests verify that the multi-class continuum model is able to reproduce the important traffic phenomena such as platoon dispersion in presence of heterogeneous users that fails in the case of homogeneous users. Further research will focus on the study of synchronized flow, cluster effects and experimental validation of this general anisotropic multi-class continuum model.

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